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MATHEMATICAL SIMULATION OF GAS TRANSPORTATION PIPELINES NETWORKS: REPETITIVE AND 2D SYSTEM THEORY SETTING

The certain classes of differential nonlinear repetitive processes and discrete 2D control systems for simulation of gas distribution networks are considered. These mathematical models are suitable for handling problems of optimal control of pressure and flow in gas transport pipeline units and in a pipeline networks. The considered problem is to optimize the total gas supply subject to some flow-pressure constraints of material balances and pressure bounds. Some aspects of a comprehensive optimization theory based on a constructive approach are discussed.

Keywords: *networks simulation; gas pipelines; repetitive processes; 2D system; control and optimization theory.*

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ГАЗОТРАНСПОРТНЫХ СЕТЕЙ: МЕТОДЫ ТЕОРИИ МНОГОШАГОВЫХ И 2D СИСТЕМ

В статье предложены методы математического моделирования распределенных газотранспортных сетей на основе многошаговых и 2D систем управления. Такие модели удобны для управления давлением и потоком газа в трубопроводах. Рассматривается задача оптимизации суммарного объема прокачиваемого газа при ограничениях на давление и поток газа. Обсуждаются некоторые аспекты построения алгоритмов оптимизации.

Ключевые слова: *математическое моделирование сетей; транспортировка газа по трубопроводам; многошаговые процессы; 2D системы управления.*

Introduction

It is well known that gas transportation networks represent complex and large scale distributed parameter system of great practical interest. Simulation approaches, numerical methods and optimization of operating modes of gas transport networks have been of permanent interest for researchers and a large number of papers were published both in civil engineering and in the mathematical investigations [1–5]. Nevertheless, optimization and control of complicated gas networks still remains a challenging problem. The general mathematical models of a gas transportation network typically include a large number of nonlinear elements such as pipelines, gasholders, compressor stations and others. In this paper the mathematical model with optimization problem of gas network units are considered on the basis of the repetitive and 2D (space and time) system theory setting. Some aspects of control theory for multidimensional systems are investigated in [6, 7] and their applications to gas networks have been considered in [8]. The main elements of constructive optimization for the repetitive processes have developed in [9, 10]. It is important to investigate problems of gas transportation for linearization models around predefined trajectories along with real-time capable well-scaled algorithms.

The method of linearization for the models of a gas delivery pipeline networks is developed in this paper and we obtain the certain classes of linear differential processes and 2D models. The problem of optimization for total supply cost of a gas transmission with the minimal guaranteed pressure at the nodes is investigated. On the base of the proposed linearization of gas pipeline model some new results in optimal control for differential linear repetitive processes with constraints are presented. The analysis is based on generalizing the well known maximum principle. There we have developed a method to establish optimality conditions in the feedback form that is of interest with theoretical point of view and applications.

Gas flow model in networks

In this section the mathematical model and corresponding optimization problem for single pipeline unit of the gas network are considered on the basis of the repetitive processes. The purpose of these modeling is to guarantee a predefined regime for pipeline unit with maximal output flow. Proposed mathematical model provides a fairly well established mathematical framework and it may be used for the further investigations of complex networks.

Gas flow model in pipeline unit. The state space parameters are gas pressure $p(t, x)$ and mass flow $Q(t, x)$ at time t and point x of the pipe, where $t \in [0, T]$ and $0 \le x \le L$. For mathematical description of the state space parameters in the case of the isothermal gas flow in a long pipeline the following system [1, 2] of nonlinear differential equations of gas dynamics may be used

$$\frac{\partial Q(t, x)}{\partial t} = -S\frac{\partial p(t, x)}{\partial x} - \frac{\lambda c^2}{2DS}\cdot\frac{Q^2(t, x)}{p(t, x)}, \quad \frac{\partial p(t, x)}{\partial t} = -\frac{c^2}{S}\cdot\frac{\partial Q(t, x)}{\partial x}, \tag{1}$$

where S — the cross sectional area; D — the pipeline diameter; c — the isothermal speed of sound; λ — the friction factor.

It is well known that some important dynamic characteristics of the processes may be received from the linearized model. The most accurate linear model may be obtained in some neighborhood of the known basic regime $\overline{Q}(t,x)$, $\overline{p}(t,x)$ of the considering process. It is shown [8], that the linearized model in some neighborhood of the known (pre-assigned, basic) regime $\overline{Q}(t,x)$, $\overline{p}(t,x)$ has the following form

$$\frac{\partial Q}{\partial t} = -S\frac{\partial p}{\partial x} - \delta Q - \beta p, \quad \frac{\partial p}{\partial t} = \alpha\frac{\partial Q}{\partial x}, \tag{2}$$

where new parameters are equal: $\delta = \dfrac{2\gamma\overline{Q}}{\overline{p}}$, $\beta = \dfrac{\gamma\overline{Q}^2}{\overline{p}^2}$, $\gamma = \dfrac{\lambda c^2}{2DS}$, $\alpha = \dfrac{c^2}{S}$.

At first, let us consider the problem for the linearized model of a single pipe.

Linear differential repetitive model for pipeline units. The linearization method, as a first order approximation, reduces the accuracy of the mathematical description of the real processes in the gas units. In order to reduce this losses some controlled inputs $r(t, x)$ and $q(t, x)$ may be added to the linear model (2) as follows

$$\frac{\partial Q}{\partial t} = -S\frac{\partial p}{\partial x} - \delta Q - \beta p + \mu r, \quad \frac{\partial p}{\partial t} = \alpha\frac{\partial Q}{\partial x} + \nu q, \tag{3}$$

where μ, ν are some normalizing coefficients. From the physical point of view, control functions $r(t, x)$ may be treated as a correcting pressure generated by compressor station and gasholders to increase the velocity $\dfrac{\partial Q(t, x)}{\partial t}$ of the running gas volume for the considered gas unit. The control variable $q(t, x)$ may be interpreted as an additional flow (supply/off take) to change the velocity $\dfrac{\partial p(t, x)}{\partial t}$ of the pressure.

Let us approximate the partial derivatives in (3) by the backward differences

$$\frac{\partial p(t, x)}{\partial x} \approx \frac{p(t, x) - p(t, x - h)}{h}, \quad \frac{\partial Q(t, x)}{\partial x} \approx \frac{Q(t, x) - Q(t, x - h)}{h}$$

and introduce the following notations:

$$Q_k(t) = Q(t, kh),\ p_k(t) = p(t, kh),\ r_k(t) = r(t, kh),\ q_k(t) = q(t, kh)$$

for the values of the unknown functions $Q(t, x)$ and $p(t, x)$ calculated at the points of lattice $\{h, 2h, \ldots, mh\} = \{kh, k = \overline{1, m}\}$. Here m is equal to the integer part of the fraction L / h and L is the length of the pipe, h is a sampling step. Then system (3) may be rewritten as follows

$$\frac{dQ_k(t)}{\partial t} = \frac{S}{h}p_{k-1}(t) - \delta Q_k(t) - \left(\frac{S}{h} + \beta\right)p_k(t) + \mu r_k(t),$$

$$\frac{dp_k(t)}{\partial t} = \frac{\alpha}{h}Q_k(t) - \frac{\alpha}{h}Q_{k-1}(t) + \nu q_k(t).$$

If we introduce the vectors $z_k(t) = \begin{pmatrix} Q_k(t) \\ p_k(t) \end{pmatrix}$, $u_k(t) = \begin{pmatrix} r_k(t) \\ q_k(t) \end{pmatrix}$ and matrixes

$$A = \begin{pmatrix} -\delta & -\frac{S}{h} - \beta \\ \frac{\alpha}{h} & 0 \end{pmatrix},\ B = \begin{pmatrix} \mu & 0 \\ 0 & \nu \end{pmatrix},$$

then the dynamical model for the pipeline unit is defined by the linear differential repetitive state space model of the form

$$\frac{dz_k(t)}{dt} = Az_k(t) + Dz_{k-1}(t) + Bu_k(t),\ k = \overline{1, m},\ t \in [0, T]. \tag{4}$$

The solution $z_k(t)$, $t \in [0, T]$ of system (4) is the output vector, or pass profile vector. The vector $z_{k-1}(t)$ produced on the previous pass $(k - 1)$ together with the control function $u_k(t)$ acts as a forcing function on the next pass k. Here we assume that the time duration period T of the process is finite. In order to complete the description of the process for a single pipeline model, it is necessary to specify the boundary and initial conditions, i.e. $z_0(t)$, $t \in [0, T]$ and $z_k(0)$, $k = \overline{1, m}$ on each pass. The boundary condition $z_0(t)$, $t \in [0, T]$ can be treated as a standard pumping regime. The initial conditions $z_k(0)$, $k = \overline{1, m}$ describe the values of this standard regime calculated in the starting moment $t = 0$ at the discrete points of lattice $x = kh$ of the pipe. In order to formulate the optimization problem it is necessary specify a cost functional. For this purpose, the problem of keeping the pre-assigned regime at the chosen points of the pipe seems reasonable. In particular, the total gas volume needs to guarantee some technically approved pressure values $p_k(T) = g_k$, $k = \overline{1, m}$ at the pre-assigned points of the pipe may be an appropriate choice. Also we may admit, that there exist several gas off-takes points which, for simplicity, coincide with some points of the lattice.

Thus, the optimal control problem can be formulated as the problem of finding the admissible controls $u_k(t)$, $t \in [0, T]$ such that the corresponding solutions $z_k(t)$, $t \in [0, T]$ of the system (4) with the given initial data keep a pre-assigned regime along the pipe at the final moment

$$p_k(T) = g_k,\ k = \overline{1, m} \tag{5}$$

and maximize the total output flow at the final moment T

$$\max_u J(u),\ J(u) = \sum_{k=1}^{m} Q_k(T), \tag{6}$$

obviously, the introduced control function $u_k(t)$, $t \in [0, T]$ not be taken arbitrarily. We assume that controls together with an ability to scale by the corresponding coefficients μ, ν may be defined as follows: for each pass number k, $k = \overline{1, m}$ the piecewise continuous function u_k: $T \to \mathbb{R}^2$ is defined as admissible control for this pass if their components $r_k(t)$, $q_k(t)$ satisfy conditions:

$$|r_k(t)| \le 1,\ |q_k(t)| \le 1,\ t \in [0, T]. \tag{7}$$

The control problem (4)–(7) gives a motivation for development of an adequate optimization method for the special classes of repetitive processes. The time-varying case of the system (4) we consider in the second part of this paper.

2D System Setting

The aim of this section is to apply the 2D control theory setting for studying control problems in gas pipeline units. For the differential system (3) we introduce the following combined sampling scheme with steps h_1, h_2 on t, x respectively:

$$\frac{\partial Q(t,x)}{\partial t} \approx \frac{Q(t+h_1,x)-Q(t,x)}{h_1}, \quad \frac{\partial Q(t,x)}{\partial x} \approx \frac{Q(t,x+h_2)-Q(t,x-h_2)}{2h_2},$$
$$\frac{\partial p(t,x)}{\partial t} \approx \frac{p(t+h_1,x)-p(t,x)}{h_1}, \quad \frac{\partial p(t,x)}{\partial x} \approx \frac{p(t,x+h_2)-p(t,x-h_2)}{2h_2}.$$

From system (3) it follows, that the discrete values $Q(k_1h_1, k_2h_2)$ and $p(k_1h_1, k_2h_2)$ of the functions $Q(t, x)$ and $p(t, x)$ calculated in the nodes of lattice $\{(k_1h_1, k_2h_2)\}$ satisfy the following equations:

$$Q((k_1+1)h_1, k_2h_2) = (1-h_1\delta)Q(k_1h_1, k_2h_2) - h_1\beta p(k_1h_1, k_2h_2) - $$
$$- \frac{Sh_1}{2h_2}\Big(p\big(k_1h_1,(k_2+1)h_2\big) - p\big(k_1h_1,(k_2-1)h_2\big)\Big), \tag{8}$$
$$p\big((k_1+1)h_1, k_2h_2\big) = p\big(k_1h_1, k_2h_2\big) + \frac{\alpha h_1}{2h_2}\Big(Q\big(k_1h_1,(k_2+1)h_2\big) - Q\big(k_1h_1,(k_2+1)h_2\big)\Big).$$

Let i, j be integers. Then with the following notations: $z(i,j) = \begin{pmatrix} Q(ih_1, jh_2) \\ p(ih_1, jh_2) \end{pmatrix}$,

$$A_0 = \begin{pmatrix} 1-\delta & -\beta \\ 0 & 1 \end{pmatrix}, \; A_1 = \begin{pmatrix} 0 & -\frac{Sh_1}{2h_2} \\ \frac{\alpha h_1}{2h_2} & 0 \end{pmatrix}, \; A_2 = \begin{pmatrix} 0 & -\frac{Sh_1}{2h_2} \\ \frac{\alpha h_1}{2h_2} & 0 \end{pmatrix}.$$

Then the system (8) can be rewritten as follows:

$$z(i+1, j) = A_0 z(i, j) + A_1 z(i, j+1) + A_2 z(i, j-1). \tag{9}$$

The mathematical model (9) is a discrete version of the gas transport network problem along the single pipe. Since each gas pipe is long, then it is reasonable to set $j \in Z_+$, where Z_+ is the set of nonnegative integers. When we apply the method of discrete approximation, the discrete values j may be huge. Let N is equal to the integer part of the fraction T / h_1, where h_1 is a sampling step for the variable t.

Then an actual problem is to find the suitable control function $u(i)$, $i = \overline{1, N}$ for the gas pressure and gas flow at the pre-defined time moments for the gas pipeline unit. Thus, we have the following 2D control optimization problem:

$$\min_u J(u), \; J(u) = \sum_{i=1}^{N}\left(\big(Ru(i), u(i)\big) + \sum_{j \in Z_+}\big(Gz(i,j), z(i,j)\big) \right), \tag{10}$$

over the solution of the system (9) with the initial and boundary control condition:

$$z(0, j) = \varphi(j), \; j \in Z_+ \setminus \{0\}, \; z(i,0) = \psi(i) = u(i), \; i = \overline{1, N}. \tag{11}$$

Here $G \geq 0$ and $R > 0$ are given symmetric matrices. The functions $u(i)$, $i = \overline{1, N}$ may be interpreted as the controlled factors: gas pressure and gas flow at the pre-assigned time moments needed to keep the desired regime how to «pump in pump out» through time. The initial data $z(0, j) = \varphi(j)$, $j \in Z_+ \setminus \{0\}$ may be treated as an pre-assigned starting pumping regime given at the initial moment $i = 0$. Also, note that the quadratic cost functional (10) is based on the estimation of the deviation from the pre-assigned regime $\overline{Q}(t,x)$, $\overline{p}(t,x)$ determined by the first-step simulation.

Boundary optimal control. An essential problem in optimization theory is to establish optimality conditions in the feedback form that is of interest in both systems theory and applications. The aim of this section is to obtain the representation of optimal control function $u(i)$, $i = \overline{1, N}$ for the optimization problem (9)–(11) by means of additional variables $w(i, j)$

that present the state variables of the so-called conjugate system. Let A'_0, A'_1, A'_2 denote the conjugate matrixes for A_0, A_1, A_2 respectively. The following result is true.

Theorem 1. *The optimal control* $u^0 = \left(u_1^0, u_2^0, \ldots, u_N^0\right)$ *of the problem (9)–(11) is given by formula*

$$u_i^0 = -R^{-1}A'_2 w^0(i,\ 0),\ i = \overline{0, N-1},$$

where $w(i,\ j)$, $j \in Z_+$, $i = \overline{0, N-1}$ *is described by the following system of equations*

$$w(i,j) = A'_0 w(i+1,j) + A'_1 w(i+1,j+1) + A'_2 w(i+1,j-1) + Q_z(i+1,j),$$

$$z(i+1,j) = A_0 z(i,j) + A_1 z(i,j+1) + A_2 z(i,j-1) - A_2 R^{-1} A'_2 w(i,0)$$

with the boundary conditions $z(0,j) = \varphi(j)$, $z(N,j) = 0$, $j \in Z_+ \setminus \{0\}$.

Some other results and details of this optimization problem are given in [8]. It is important to investigate such problems on conditions of uncertainty, for example, with fuzzy description of noise and perturbations. The application of concepts and methods of fuzzy set theory to problems for linear 2D systems is given in [11].

Constrained optimization of time-varying repetitive processes

Here we develop the method to establish optimality conditions in the classic form for a particular case of differential repetitive processes with nonlinear inputs and nonlocal state-phase terminal constraints of general form. The problem statement in the proposed form follows from the mathematical simulation of the distributed gas network given above. In order to extend the proposed model we will consider a general convex case of objective cost functional and state constrains of general form at the given time moments. Also, we consider the model with matrixes as functions of the temporary variables.

Let $[0,\ T]$ be a given interval of values of the continuous independent variable t and $K = \{1, 2, \ldots, N\}$, $N < +\infty$ be the set of values of the discrete variable k. Let us introduce the control and state vectors as $u_k(T) \in \mathbb{R}^m$ and $y_k(t) \in \mathbb{R}^n$ respectively. Then the considered repetitive processes are described by

$$\frac{dy_k(t)}{dt} = A(t)y_k(t) + D(t)y_{k-1}(t) + b_k(u_k(t), t),\ k \in K,\ t \in [0, T], \tag{12}$$

where the last nonlinear term represents the input signal actually applied to the process. To complete the description, it is necessary to specify the boundary conditions which are here taken to be of the form

$$y_k(0) = \eta(k),\ k \in K,\ y_0(t) = \xi(t),\ t \in [0, T]. \tag{13}$$

Now we define the class of available and admissible controls. We say that the function u: $K \times [0, T] \to \mathbb{R}^m$ is available for system (12) if it is measurable with respect to t for fixed $k \in K$ and satisfies the constraint $u_k(t) \in U$ for almost all $t \in [0, T]$, where U is a given compact set from $\mathbb{R}^m$. Also the function y: $K \times [0, T] \to \mathbb{R}^n$ is a solution of system (12) corresponding to the given available control $u_k(t) \in U$ if it is absolutely continuous with respect to $t \in [0, T]$ for each fixed $k \in K$ and satisfies to system (12) for almost all $t \in [0, T]$ and each $k \in K$. We denote the set of available controls by Ω and use M_i, $M_i \subseteq \mathbb{R}^n$, $i = 1, 2, \ldots, \vartheta$ to denote the given compact convex sets. The available control $u_k(t) \in U$ is said to be admissible for the system (12) if the corresponding solution $y_k(t) = y_k(t, \eta, \xi, u)$ of system (12) and boundary conditions (13) satisfies to the given constrains $y_N(\tau_i) \in M_i$, $i = 1, 2, \ldots, \vartheta$ where $0 < \tau_1 < \tau_2 < \ldots < \tau_\vartheta$ are specified elements of the time segment $[0,\ T]$.

The optimal control problem may be stated as: minimize the cost functional

$$\min_{u \in \Omega} J(u),\ J(u) = \Phi\left(y_N(\tau_1), y_N(\tau_2), \ldots, y_N(\tau_\vartheta)\right) \tag{14}$$

for processes described by system (12) and boundary conditions (13) in the class of admissible controls $u \in \Omega$. We also assume that the $n \times n$ matrix functions $A(t)$ and $D(t)$ and the n-vector function $\xi(t)$ are measurable and integrable on [0, T], the function b: $K \times U \times [0,T] \to \mathbb{R}^n$ is continuous with respect to $(u,t) \in U \times [0,T]$ for each fixed $k \in K$ and the function $\Phi: \mathbb{R}^{n\vartheta} \to \mathbb{R}$ is convex. It is easy to see that these conditions guarantee the existence and uniqueness of an absolutely continuous solution of system (12), (13) for any available contro $u_k(t)$. To guarantee the existence of optimal control we assume that the set of admissible controls is non-empty. Examples of such systems (12)–(14) include robotic manipulators that are required to repeat a given task to high precision, chemical batch processes or, more generally, the class of tracking systems and others. Some results and details concerning the considered processes described by system (12), (13) may be found in [9].

Optimality conditions. To formulate the optimality conditions it is necessary to introduce some functions. Let us consider function $\psi: K \times [0,T] \to \mathbb{R}^n$

$$\psi_k(t) = \int_0^t F'(\tau,t)D'(\tau)\psi_{k-1}(\tau)d\tau,\ k \in \{2,3,\ldots,N\},\ \psi_1(t) = \gamma(t),$$

where the function $\gamma(t)$ is the solution of the linear differential equation

$$\frac{d\gamma(t)}{dt} = -A'(t)\gamma(t),\ t \in [0,T]$$

with jump conditions $\gamma(\tau_j-) - \gamma(\tau_j+) = g_j^0$, $j = 1,2,\ldots,\vartheta-1$. Here $g^0 = (g_1^0, g_2^0, \ldots, g_\vartheta^0)' \in \mathbb{R}^{n\vartheta}$ is the maximizing vector for the smallest root ε^0 of the equation

$$\Lambda(\varepsilon) = 0,\ \Lambda(\varepsilon) = \max_{\|g\|=1}\{g'c - \max_{\omega\in X(\varepsilon)} g'\omega + \max_{u\in\Omega} g'\Pi u\}. \tag{15}$$

Here the set $X(\varepsilon)$ is defined as $X(\varepsilon) = \{\omega \in \mathbb{R}^{n\vartheta}: \omega \in M, \Phi(\omega) \le \varepsilon\}$ where $M = M_1 \times M_2 \times \ldots \times M_\vartheta \subseteq \mathbb{R}^{n\vartheta}$ and ε is a number from $\mathbb{R}$. The mapping $\Pi: \Omega \to \mathbb{R}^{n\vartheta}$ is the vector valued function $\Pi u = (\Pi_1 u, \Pi_2 u, \ldots, \Pi_\vartheta u)'$ with component

$$\Pi_j u = \sum_{i=1}^{N-1} P^i Q b_{N-i}\big(u_{N-i}(\tau_j),\tau_j\big) + \int_0^{\tau_j} F(\tau_j,t)b_N\big(u_N(t),t\big)dt,\ \ j = 1,2,\ldots,\vartheta,$$

where the mappings involved are as follows:

$$Qf(\tau) = \int_0^\tau F(\tau,t)f(t)dt,\ \ Pf(\tau) = \int_0^\tau F(\tau,t)D(t)f(t)dt,\ \ \tau \in [0,T],$$

with its power composition $(P^k f)(\tau) = P(P^{k-1}f)(\tau), \tau \in [0,T]$ The vector $c = (c_1, c_2, \ldots, c_\vartheta)' \in \mathbb{R}^{n\vartheta}$ is given as follows:

$$c_j = F(\tau_j,0)\eta(N) + \sum_{i=1}^{N-1}\Big(P^iF(\cdot,0)\Big)(\tau_j)\eta(N-i) + \Big(P^N\xi\Big)(\tau_j),\ \ j = 1,2,\ldots,\vartheta.$$

The $n \times n$ matrix function $F(\tau,t)$ is the solution of the differential equation: $\dfrac{\partial F(\tau,t)}{\partial\tau} = A(\tau)F(\tau,t)$, $F(t,t) = E_n$, where E_n denotes the $n \times n$ identity matrix.

The optimality conditions for (12)–(14) are given by the following theorem.

Theorem 2. *If the number ε^0 is the smallest root of the equation (15), then there exists an optimal control $u_k^0(t), k \in K, t \in [0,T]$ for the problem (12)–(14) such that* $J(u^0) = \varepsilon^0$ and *for almost all $t \in [0,T]$ the minimality principle (minimality conditions)*

$$\psi'_k(t)b_{N-k+1}(u^0_{N-k+1}(t),t) = \min_{\sigma\in U}\psi'_k(t)b_{N-k+1}(\sigma,t)$$

for all $k \in K$ are fulfilled.

It should be noted, that the most of real complex and large scale distributed gas networks may be described by time-varying repetitive models with quasidifferentiable coefficients [12].

Conclusion

This paper presents some results on mathematical description of the distributed gas networks in framework of multistage simulation. For this purpose mathematical analysis of distributed gas networks in the framework of a repetitive processes and 2D system theory is given. At first, we present the mathematical model of network pipe units based on the stationary repetitive differential linear systems. That gives us a good tool to express potentially critical flow/pressure values within the certain margins in order to optimize the demand distributed over single pipe units. Optimal control problem for stationary differential linear repetitive processes has investigated on the base of supporting control functions approach [9]. The main contribution here is the development of constructive necessary and sufficient optimality conditions that may be used for create numerical algorithms. Farther, we use the linear-quadratic optimization approach for 2D control system. The method to establish optimality conditions in the feedback form have developed. Further research should aim to extend the presented models to complicated gas networks, which are more relevant for applications.

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APPROACHES TO ASSESSMENT OF COMPETITIVENESS OF NETWORK TRADING BUSINESS

The article considers approaches to determining the essential characteristics of the network business competitiveness, methods of its assessment, a system of indicators and analysis criteria, which will allow a more balanced approach to the development of strategies and tactics of competition and ensure long-term retention of competitive advantages.

Keywords: *competitiveness; competitive advantages; network trading business; assessment; approaches; criteria; indicators.*

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ПОДХОДЫ К ОЦЕНКЕ КОНКУРЕНТОСПОСОБНОСТИ СЕТЕВОГО ТОРГОВОГО БИЗНЕСА

В статье рассматриваются подходы к определению сущностных характеристик конкурентоспособности сетевого бизнеса, методов ее оценки, системы показателей и критериев анализа, что позволит более взвешенно подойти к разработке стратегии и тактики конкурентной борьбы, обеспечить длительное удержание конкурентных преимуществ.

Ключевые слова: *конкурентоспособность; конкурентные преимущества; сетевой торговый бизнес; оценка; подходы; критерии; показатели.*

Введение. Развитие экономики в Республике Беларусь сопровождается быстрым внедрением информационных технологий, виртуальных сред, электронных пространств, цифровых платформ, позволяющих воспроизводить и наращивать стоимость в любом территориальном, временном и пространственном сегменте. Включение этих составляющих в цепочки производства стоимости (в том числе и добавленной) активно вовлекает и потребителя в систему производства стоимости, масштабирует связи, формирует их иерархию и выдает их в виде различных сетей, в той или иной мере связанных между собой с различной степенью тесноты, времени взаимовыгодного функционирования. В этой ситуации активные действия по поиску производителей, потребителей,