# ON REARRANGEABLE NETWORKS WITH AT MOST ONE RESWITCHING

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## ABSTRACT

It is well known that (applying usual notations according to fig. 1) three stage connecting networks with $m \geq r$ are non-blocking, if the method of rearrangement is applied, and that at most $r-1$ reswitchings are necessary. Paull [4] stated that the number of reswitchings may be reduced by increasing the number $m$ of middle switches. He showed that for networks with $r=n$ and $m=2n-2$, the reswitching of one connection is always sufficient.

In the present paper it is shown that for networks with $m=2n-2$, one reswitching is always sufficient if $r \leq 2n-2$. Furthermore it is show that one reswitching is also sufficient in particular networks with $m<2n-2$. The results of this paper are proved by means of simple considerations and illustrated by examples.

## 1. INTRODUCTION

This paper deals with switching networks with rearrangement (or, rearrangeable networks) [2, 3, 4, 5] in which calls in progress (which will be denoted as connections in the sequel) may be reswitched. Recently such systems (in the form of networks with rearrangement) have been realized, e.g., in so called cross connects.

In the usual way, a three stage switching network as shown in fig. 1 is denoted as $(m, n, r)$ network. There are $r$ switches $(n \times m)$ in the first stage, $m$ switches $(r \times r)$ in the second stage and $r$ switches $(m \times n)$ in the third stage, which, for reasons of simplicity, will be denoted as input switches, middle switches and output switches, respectively, in the sequel. As is well known, $(n, n, r)$ networks as shown in fig. 2 are rearrangeable [5] whereas $(2n-1, n, r)$ networks (Clos networks) are non-blocking in the strict sense [1].

As is well known, the reswitching procedure of $(n, n, r)$ networks requires a maximum of $r-1$ reswitchings. [2, 3, 4, 5] A reswitching algorithm which is suitable for practical applications has been given by Paull in [4] for the special case of $(n, n, n)$ networks. This algorithm is, however, also applicable for other values of $r$ $(r \neq n)$ [6, 7].

The number of necessary reswitchings can be reduced by increasing the number $m$ of middle switches. Paull has shown that for networks with $r=n$ and $m=2n-2$, one reswitching is always sufficient [4].

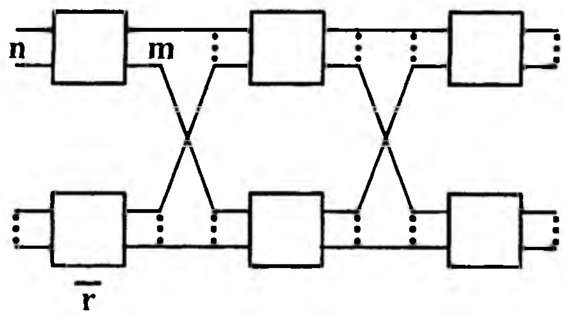


*Figure 1.* $(m, n, r)$ switching network

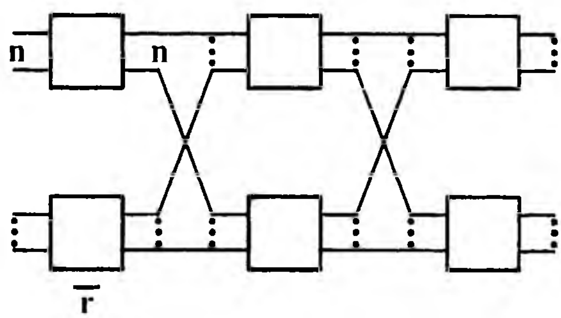


*Figure 2.* Rearrangeable $(n, n, r)$ network

In the present paper, networks are considered in which one reswitching is always sufficient. Section 2 deals with networks with $m=2n-2$ middle switches. It is shown that in this case the maximum number of reswitchings is one, if (and only if) $r \leq 2n-2$ (i.e., $r \leq m$) switches in the first and third stages are existing. In section 3 networks are investigated where the number of middle switches is $m<2n-2$. It is shown that for particular networks of this type one reswitching is also sufficient. A short overview of these results is given in section 4. After a consideration of the required number of cross points in section 5, a conclusion follows in section 6. The results of this paper are proved by means of simple considerations and illustrated by means of examples.

## 2. NETWORKS WITH $m=2n-2$ MIDDLE SWITCHES

### 2.1. Networks with $m=2n-2$ Middle Switches and $r \geq n$ Input and Output Switches

#### 2.1.1. Basic Considerations

The switches of each stage be numbered, which leads to the notations $I_j$ $(j=1...r)$ for the input switches, $M_k$ $(k=1...m)$ for the middle switches and $O_p$ $(p=1...r)$ for the output switches (i.e., the number of an input switch, middle switch or output switch is denoted by $j$, $k$ and $p$, respectively).

The general idea in the following investigations is to consider a blocked call and then find out how – in a worst case – the network can be reswitched such that the new call can be switched. Without loss of generality it is supposed that the new call has to be switched between the first input switch ($I_1$) and the first output switch ($O_1$). Calls between other switches can be mapped to this case by a suitable renumbering of the input and output switches. Analogously, in case of equivalence, other occupation patterns (and occupation patterns considered in the sequel) can be mapped to the corresponding patterns considered here by renumbering of switches.

It is supposed that a new call from input switch $I_1$ to output switch $O_1$ is feasibly (possibly after a rearrangement procedure), implying that at least one input of $I_1$ and one output of $O_1$ are idle. Furthermore it is supposed that this call is blocked, i.e., there exists no middle switch with an idle link to $I_1$ as well as an idle link to $O_1$. This blocking situation is supposed to represent a worst case (with respect to the switching of the new call). Such a worst case [5] is existing if, beside the idle input of the input switch $I_1$ where the new call arrives, all other $n-1$ inputs of $I_1$ are busy, and in the same way, beside the output of $O_1$ for the new call, all other $n-1$ outputs of $O_1$ are busy. Therefore a blocking state of the network (for a new call from $I_1$ to $O_1$) must have the form of the state shown in fig. 3. Here, the existing connections via input switch $I_1$ and via output switch $O_1$ are all switched via different middle switches. In the sequel a blocking state according to fig. 3 will be used as a basis for further considerations.

For the further continuation of the existing connections indicated in fig. 3, there are two possible cases:

a) All existing connections via input switch $I_1$ go to different output switches $O_p$, and all existing connections via $O_1$ go to different input switches $I_j$. This case will be denoted as "regular distribution of the existing connections". It should be pointed out that due to the fact that all existing connections via $I_1$ and $O_1$ are using different middle switches (as indicated in fig. 3), it is not possible in a blocking state in case of $m=2n-2$, that existing connections are switched between $I_1$ and $O_1$.

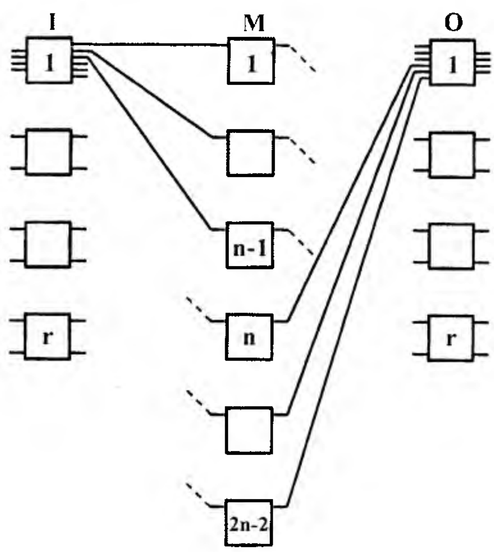


*Figure 3.* Network state in which a new call $I_1O_1$ is blocked

b) The existing connections via input switch $I_1$ do not all go to different output switches $O_p$, i.e. there is at least one output switch $O_p$ $(2 \leq p \leq r)$ with more than one connection leading to input switch $I_1$, and/or the existing connections via $O_1$ do not all lead to different input switches $I_j$ $(2 \leq j \leq r)$. This case will be referred to as "irregular distribution of existing connections".

#### 2.1.2. Regular distribution of the existing connections

In case of a regular distribution of all existing connections via $I_1$ and $O_1$ leading to a state of the network as shown in fig. 4, it can be seen that an existing connection via input switch $I_1$ is switched via middle switch $M_k$ $(1 \leq k \leq n-1)$ and an output switch $O_p$ $(2 \leq p \leq n$, where $k=p-1)$. The route of such a connection is $I_1$, $M_{p-1}$, $O_p$. From fig. 4 it can be seen that all links between $I_1$ and $M_k$ $(n \leq k \leq 2n-2)$ are idle. If a link between $M_k$ and $O_p$ $(n \leq k \leq 2n$-$2$, $2 \leq p \leq n)$ is idle (e.g., for $k=k'$ and $p=p'$), then the existing connection from $I_1$ to $O_{p'}$, i.e. the connection $I_1$ $M_{p'-1}$ $O_{p'}$ can be reswitched to $I_1$, $M_{k'}$ $O_{p'}$. The new connection can then be switched via $I_1$ $M_{p'-1}$ $O_1$.

Between the switches $O_p$ ($2 \leq p \leq n$) and $M_k$ ($n \leq k \leq 2n-2$) each, one link is existing, respectively, which is in principle suitable for a reswitching of one of the existing connections via $I_1$. In total these are $(n-1)^2$ links. If one of these links is idle, the reswitching of a connection via $I_1$ is possible, and thus the reswitching of one existing connection is sufficient for switching the new call. Analogously, the links between the switches $I_j(2 \leq j \leq n)$ and $M_k$ ($1 \leq k \leq n-1$) can be used for the reswitching of an existing connection via output switch $O_1$. These are again $(n-1)^2$ links which are suitable for the reswitching of the connection via $O_1$.

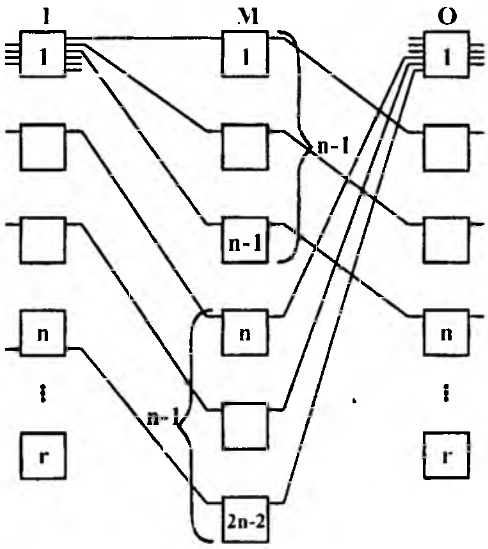


*Figure 4.* System state with regular distribution of the existing connections via $I_1$ and $O_1$

It can easily be seen that the middle switches suitable for reswitching of an existing connection via input switch $I_1$ are different from those middle switches which are suitable for reswitching an existing connection via output switch $O_1$. Thus there exists no route between input switch $I_1$ and output switch $O_1$ containing a link suitable for reswitching of an existing connection via $I_1$ as well as a link suitable for reswitching an existing connection via $O_1$.

$$N_{links}=2(n-1)^2. \qquad (1)$$

Some (or all) of these links may be blocked by additional connections. Because, as stated above, there exist no routes containing two of these links for additional connections, each additional connection can block only one of these links which are suitable for a reswitching. “Additional” connections in this sense comprise arbitrary connections with the exception of the $(n-1)$ existing connections via input switch $I_1$, the $(n-1)$ existing connections via output switch $O_1$ and the new call. In total at most $nr$ connections are possible. Therefore, the maximum number of additional connections, which be denoted as $N_{add}$, amounts to

$$N_{add}=nr-2(n-1)-1. \qquad (2)$$

A reswitching as explained above and thus the switching of the new call is possible if and only if at least one of the $N_{link}$ links which are suitable for reswitching is idle (i.e., not occupied by an additional connection), i.e.,

$$N_{add}<N_{link} \qquad (3a)$$

or, as $N_{add}$ and $N_{link}$ are in integral numbers, if

$$N_{add} \leq N_{link}-1 \qquad (3b)$$

Inserting equations (1) and (2) yields

$$nr-2(n-1)-1 \leq 2(n-1)^2-1$$

or, after rearranging terms and, finally, after dividing by $n$ (where $n \neq 0$)

$$r \leq 2n-2. \qquad (4)$$

This leads to the following result:

In networks with $m=2n-2$ and $n \leq r \leq 2n-2$, the reswitching of one connection is always sufficient.

(In case $r \geq 2n-1$, the reswitching of only one connection is not always sufficient for enabling the switching of a new call.)

### 2.1.3. Irregular Distribution of the Existing Calls

In case of an irregular distribution of the existing connections via input switch $I_1$ there exists at least one output switch $O_p$ (with $p \geq 2$) carrying two or more of the existing connections via input switch $I_1$. This is indicated in fig. 5. The number of this particular output switch be $p'$ and the number of existing connections via input switch $I_1$ which are carried by this output switch $O_{p'}$ be denoted by f (where $2 \leq f \leq n-1$).

First the case $f=2$ be considered. In this case $f=2$ outputs of the $n$ outputs of this output switch $O_{p'}$ are occupied by existing connections via input switch $I_1$. Additional connections via output switch $O_{p'}$ may exist which are blocking links from output switch $O_{p'}$ to middle switches $M_k$ which are suitable for reswitching a connection (from input switch $I_1$ to output switch $O_{p'}$, as stated above). The maximum possible number of such additional connections via output switch $O_{p'}$ be denoted as $n_{add}$, and the number of links (between output switch $O_{p'}$ and middle switches $M_k$) which are suitable for reswitching an existing connection between $I_1$ and $O_{p'}$ be denoted as $n_{link}$. In analogy to section 2.1.2, the number $n_{link}$ of link s of output switch Op' which are suitable for a reswitching amounts to

$$n_{link}=n-1. \quad (6)$$

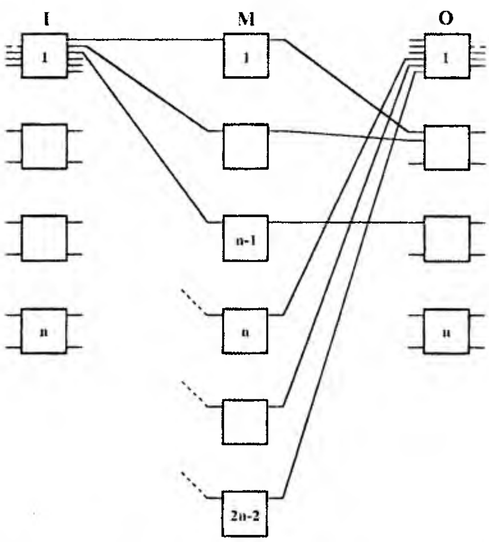


*Figure 5.* Principle of an irregular distribution of existing connections via input switch $I_1$

As f of the $n$ outputs of output switch $O_{p'}$ are occupied by connections to input switch $I_1$, the maximum number $n_{add}$ of additional connections via output switch $O_{p'}$ is

$$n_{add}=n-f \quad (7a)$$

or, for $f$=2,

$$n_{add}=n-2. \quad (7b)$$

As the number $n_{link}=n-1$ of links usable for reswitching a connection is larger than the maximum number $n_{add}$ of additional connections via output switch $O_{p'}$, it is not possible that all links suitable for reswitching are occupied by additional connections. Instead there is at least one link idle which is suitable for reswitching a connection.

In the general case of $f\geq2$ existing connections between input switch $I_1$ and output switch $O_{p'}$ holds

$$n_{add}=n-f\leq n-2.$$

Regarding equation (6) yields

$$n_{add}<n_{link}. \quad (8)$$

Hence it is always possible to reswitch an arbitrary one of these connections between $I_1$ and $O_{p'}$ in order to enable the switching of the new connection. Analogous arguments apply if there is an irregular distribution of the existing connections via output switch $O_1$.

Thus in networks with $m$=2$n$–2 middle switches, it is also possible in case of an irregular distribution of the existing connections via input switch $I_1$ and/or output switch $O_1$ to switch a new call after reswitching at most one existing connection.

Taking into account the results of section 2.1.2, it can be seen that for networks with $m$=2$n$–2 middle switches and $n\leq r\leq 2n-2$ input switches and output switches, respectively, it is always possible to switch a new call after reswitching at most one existing connection.

### 2.1.4. Networks with *m*=2*n*–2 Middle Switches and *r*<*n* Input and Output Switches

This section deals with networks where the number $r$ of input switches and output switches, respectively, is less than the number $n$ of inputs or outputs, or

$$r\leq n-1. \quad (9)$$

In section 2.1.3 it has been shown that in a considered network state under worst case conditions there are $n$–1 connections existing which are switched via input switch $I_1$, and that these connections go to output switches $O_p$ where $2\leq p\leq r$. I.e., for these $n$–1 connections there are only $r$–1 output switches which can be used for these connections in the third stage. Due to equation (9) holds

$$r-1\leq n-2.$$

From this it follows that in networks with $r<n$ input switches and output switches, respectively, there is always an irregular distribution of existing calls via input switch $I_1$ if the network is in a worst case condition, and, analogously an irregular distribution of the existing connections via output switch $O_1$.As has been shown in section 2.1.3, in network states with irregular distribution of the existing connections via input switch $I_1$ and/or output switch $O_1$, the new call can always be switched after reswitching at most one existing connection via input switch $I_1$ or output switch $O_1$.

## 2.2. Maximum Number of Reswitchings in Networks with *m*=2*n*–2 Middle Switches

From the investigations in the sections 2.2 and 2.3, the following results can be drawn:

In networks with $m$=$n$–2 middle switches, the reswitching of at most one existing connection is always sufficient for enabling the switching of a new call if and only if the number r of input switches and output switches, respectively, is at most $r \leq 2n-2$ or, $r \leq m$( if $r>2n-2$, the maximum number of connections which must be reswitched in order to enable the switching of the new call is larger than 1).

In analogy to the theory of Clos systems [1], individual pairs of an inlet and an outlet can be cancelled including the crosspoints which are connected directly to these inlets and outlets. In this way it is possible to construct switching networks where the total number $N$ of inlets (and outlets) is not exactly divisible by the number $n$ of inlets of an inlet switch or of outlets of an outlet switch, respectively. For each omitted pair of an inlet and an outlet, the total number of crosspoints is reduced by $2m$. This also applies for the networks presented in section 3. Networks constructed in this way will be denoted as "reduced networks" in the sequel.

For the well known $(n, n, r)$ switching networks with rearrangement, (requiring at most the reswitching of $r-1$ connections) Paull [4] has suggested the use of an additional middle switch in order to avoid disturbing calls in progress (i.e., connections) during the reswitching procedure. (Another reason for the use of such an additional middle switch can be seen in the fact that in many systems it is not possible to reswitch several connections at the same time but only one at a time, i.e., sequentially [7]). Such an additional middle switch enables moving a call in such a way that first a new connection is set up in parallel to the existing one and then the original connection is released.

In case of the networks considered here with the reswitching of at most one connection, such an additional middle switch is not necessary. Nevertheless, in these networks it is possible to set up the new connection first in parallel to the existing one and then to release the original connection. Thus, in the networks considered here the reswitching of a connection can be carried out without disturbing the concerned call in progress. This also applies for the networks considered in section 3.

Crosspoints requirements and examples will be considered in section 5.

## 3. Networks With $M<2N-2$ Middle Switches And At Most One Reswitching

### 3.1. General Aspects

This section deals with a new type of networks with $m<2n-2$ middle switches and with reswitching of at most one connection. The number $m$ of middle switches can then be represented in the form

$$m=2n-1-g \tag{10a}$$

or

$$g=2n-1-m. \tag{10b}$$

Thus the parameter g is the difference of the number of middle switches m of the considered network with at most one reswitching and a Clos network (which is strictly non-blocking, without rearrangement, with $2n-1$ middle switches). The special case $g=1$, i.e., $m=2n-2$, has already been discussed in section 2. Therefore this section deals only with the case

$$g \geq 2. \tag{11}$$

In section 2 it has been shown that for $m=2n-2$ the $n-1$ existing connections via input switch $I_1$ and the $n-1$ existing connections via output switch $O_1$ (under worst case conditions) are all using different middle switches. This is possible as there are in total $2(n-1)$ of connections of this type and $2n-2$ middle switches. In networks with $m<2n-2$ middle switches as considered here this is not possible. Instead there must be at least one middle switch carrying one existing connection via input switch $I_1$ and one existing connection via output switch $O_1$. The general case is indicated in fig. 6.

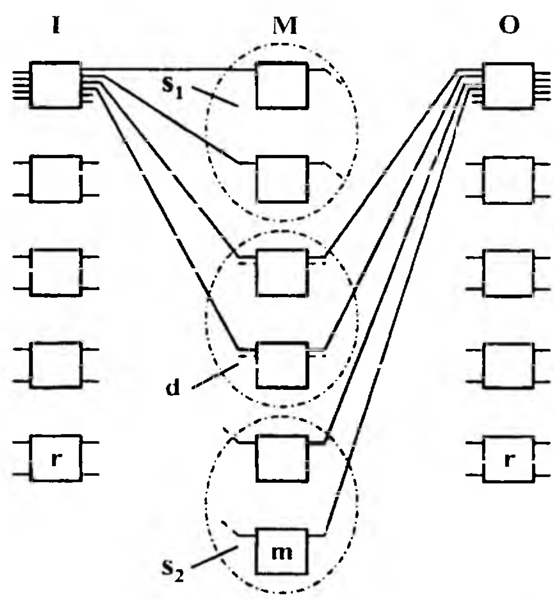


*Figure 6.* Existing connections in networks with $g \geq 2$

This leads to the following conclusion: In total there are $2(n-1)^2$ links which are suitable for reswitching a connection. This number of links be denoted as $N_{links}$, i.e.

In order to guarantee that a considered new call from input switch $I_l$ to output switch $O_l$ is blocked, each middle switch must carry at least one existing connection to either input switch $I_l$ or output switch $O_l$. I.e., according to fig. 6 there are three possible states of middle switches: Middle switches with only an existing connection to input switch $I_l$, middle switches with only an existing connection to output switch $O_l$, and middle switches with an existing connection to input switch $I_l$ as well as an existing connection to output switch $O_l$.

The number of middle switches carrying only an existing connection to input switch $I_1$ (i.e., only the "single" connection to input switch $I_1$) be denoted by $s_1$, the number of middle switches carrying only a ("single") connection to output switch $O_1$ be denoted by $s_2$. The number of middle switches carrying two existing connections (to input switch $I_1$ and to output switch $O_1$, i.e. middle switches with "double" occupancy due to existing connections) be denoted by $d$. For symmetry reasons the values $s_1$ and $s_2$ are identical, and this common value will be denoted by $s$:

$$s_1=s_2=s. \qquad (12)$$

The fact that the total number of middle switches is $m$ yields the relation

$$m=d+2s. \qquad (13)$$

Furthermore the number of existing connections via input switch $I_1$ and output switch

$O_1$ equals $2(n-1)$. This yields

$$2(n-1)=2s+2d$$

or

$$n-1=d+s. \qquad (14)$$

The relations (13) and (14) lead to the following *equations*:

$$d=2n-2-m, \qquad (15)$$

$$s=m-n+1. \qquad (16)$$

*Inserting* equation (10a) yields

$$d=g-1, \qquad (17)$$

$$s=n-g. \qquad (18)$$

### 3.2. Reswitching Aspects

Considering the reswitching of an existing connection via input switch $I_1$, it is obvious (according to fig. 6) that these connections can only be reswitched to middle switches to which there are idle links from input switch $I_1$, i.e., the $s_2$ middle switches carrying no connections via input switch $I_1$. For reswitching a connection from input switch $I_1$ to an output switch $O_p$, only the links to these $s_2$ middle switches are suitable. Thus due to equation (10), from an output switch $O_p$ there are only $s$ links (to the $s_2$ middle switches) which are suitable for reswitching of a connection between input switch $I_1$ and output switch $O_p$. Thus the number $n_{link}$ of links suitable for reswitching of a connection from input switch $I_1$ to output switch $O_p$ is in this case

$$n_{link}=s \qquad (19)$$

As stated above, only such connections via input switch $I_1$ are suitable for reswitching which use one of the $s_1$ middle switches but not one of the $d$ middle switches. Such connections will be referred to as "reswitchable" in the sequel.

### 3.3. Worst Case Conditions

It can easily be seen that the worst case is given by a network state where each of the output switches (except $O_1$) carries at least one reswitchable connection. In such a network state it is of interest, how many connections via input switch $I_1$ must exist in order to make sure that among the $r-1$ output switches there is at least one output switch carrying at least a given number $f$ of these connections. Obviously, if the total number $n-1$ of these connections is equal to $(f-1)$ $(r-1)$, then these connections can be distributed such on the output switches that each of the $r-1$ output switches carries exactly $f$-1 connections. In this case no output switch with $f$ connections would occur. By one further connection, however, it is guaranteed that at least one of these output switches carries at least $f$ connections. Thus in order to guarantee that there is at least one output switch with at least $f$ connections, it is necessary that

$$n-1>(f-1)\,(r-1). \qquad (20)$$

The reswitching of an existing connection via input switch $I_1$ is always possible if at least one of the $n_{link}=s$ links according to equation (19) which are suitable for a reswitching is idle. This is the case if, according to equation (8), a number $n_{add}$ of possible additional connections in the considered output switch is less than the number $n_{link}$ of suitable links, or, with equations (7a) and (19)

$$n-f<s. \qquad (21)$$

As stated above this is the case if the number $n-1$ of existing connections via input switch $I_1$ fulfils the equation 20. From equation (21) follows

$$f>n-s \qquad (22)$$

or

$$f\geq n-s+1. \qquad (23)$$

Combining equations (20) and (24) yields

$$n-1\geq(n-s)(r-1)+1 \qquad (24)$$

or, regarding equation (18),

$$r \leq (n-2)/g+1. \qquad (25)$$

(It can easily be shown that a different configuration, where one output switch carries all connections via input switch $I_1$ which are not reswitchable, whereas the reswitchable connections are carried on the remaining output switches does not represent a worst case and therefore need not be considered.)

This leads to the following conclusion: *A network with $m=2n-1-g$ (with $g \geq 2$) is rearrangeable with reswitching of at most one connection if and only if $r \leq (n-2)/g+1$.*

## 4. Results

In total sections 2 and 3 yield the following results. A three-stage switching network is rearrangeable by means of reswitching of at most one existing connection if and only if $r$ fulfils the condition

$$r \leq r_{max} \qquad (26)$$

In case of $m=2n-2$ holds

$$r_{max}=2n-2. \qquad (27)$$

and in case of networks with $n=2n-1-g$ middle switches (with $g \geq 2$) holds

$$r_{max}=(n-2)/g+1. \qquad (28)$$

## 5. Aspects Of Cross Point Requirements And Examples

### 5.1. Crosspoint Requirements

Using suitable existing technologies, it is nowadays in many cases possible to realise fairly cheap crosspoints, and the cross point requirements of networks can often be considered to be of minor relevance. Nevertheless, it seems that some considerations of cross point requirements are appropriate.

For the considered switching networks, it is of interest which values of $r$ lead to most advantageous networks in terms of cross point requirements. In case of networks treated in section 2, this leads to the question for which values of $r$ these networks with the parameters $n$ and $m=2n-2$ are optimal with respect to cross point requirements.

Due to the fact that $r$ and $n$ are integral values, a detailed investigation of this kind would be rather lengthy. However, detailed investigations for Clos networks (which are strictly non-blocking without the necessity of rearrangement) are well known [1]. For a given value of $n$, the corresponding value $m=2n-1$ for Clos networks [1] is very close to the value $m=2n-2$ of the networks considered here. Therefore it can be expected that the optimum values of $r$ in case of rearrangeable networks considered here will tend to be roughly in the same range as in case of Clos networks.

From the investigation of Clos networks [1] it is known that in case of optimum values of $r$ the quotient $r/n$ is roughly in the range of about 2 to 3, on the average a little larger than 2. Therefore it can be concluded that in the networks considered here the optimum values of $r$ will be roughly in the same range. I.e., these optimum values of $r$ will tend to be about $2n$ or a little larger, and it seems advisable to try to choose the values of $r$ accordingly in the case of the networks considered here. In the networks investigated in section 3, the maximum admissible value of $r$ is $r_{max}=2n-2$. Thus it follows that, aiming at networks with low cross point requirements, it is advisable to choose this maximum value $r_{max}=2n-2$ or smaller values which are as close as possible to $r_{max}$.

For the networks presented in section 3, the values of $r$ are in all cases at most $n/2$ or less. This is a disadvantage of these networks concerning the cross point requirements.

Usually networks with rearrangement will only be of interest (if so at all) if the cross point requirement is less than in case of a Clos network or a single stage matrix, i.e., a square array, (which are non-blocking without the necessity of rearrangements) with the same number of inlets and outlets. Other networks may only be of theoretical interest.

### 5.2. Examples

#### 5.2.1. Networks With $m=2n-2$ According To Section 2

In the sequel, only networks with $r>2$ are considered. The number of required cross points of a network is denoted by $CP$. The smallest networks of this type are obtained for $n=3$:

$$n=3,\ m=4,\ r=4,\ N=13,\ CP=170$$

$$n=3,\ m=4,\ r=3,\ N=10,\ CP=118$$

In both cases more crosspoints are required than in case of a single stage matrix.

In tabl. 1 some networks are listed requiring less crosspoints than a one stage matrix or a Clos network. (The remark "reduced" in this table indicates that the network concerned is a reduced network). Tabl. 2 shows some examples of larger networks of this kind.

*Table 1*. Examples of networks with $m$=2$n$–2

| n | m | r | N | CP | matrix | Clos network | Remark |
|---|---|---|---|---|---|---|---|
| 4 | 6 | 6 | 24 | 504 | 576 | 560 | |
| 4 | 6 | 6 | 23 | 492 | 529 | | Reduced |
| 4 | 6 | 6 | 22 | 480 | 484 | | Reduced |
| 4 | 6 | 5 | 20 | 390 | 400 | | |
| 5 | 8 | 8 | 40 | 1152 | | 1260 | |
| 5 | 8 | 8 | 39 | 1136 | | 1220 | Reduced |
| 5 | 8 | 8 | 38 | 1120 | | 1180 | Reduced |
| 5 | 8 | 8 | 37 | 1104 | | 1140 | Reduced |
| 5 | 8 | 7 | 35 | 952 | | 1033 | |
| 5 | 8 | 7 | 34 | 936 | | 995 | Reduced |
| 5 | 8 | 7 | 33 | 920 | | 935 | Reduced |
| 5 | 8 | 6 | 30 | 768 | | 800 | |
| 5 | 8 | 6 | 29 | 752 | | 766 | Reduced |
| 5 | 8 | 5 | 25 | 600 | | 609 | |
| 6 | 10 | 10 | 60 | 2200 | | 2376 | |

*Table 2*: Further examples of networks with $m$=2$n$–2

| *n* | *m* | *R* | *N* | *CP* |
|---|---|---|---|---|
| 10 | 18 | 18 | 180 | 12 312 |
| 16 | 30 | 30 | 480 | 55 800 |
| 20 | 38 | 38 | 760 | 112 632 |
| 32 | 62 | 62 | 1984 | 484 344 |
| 50 | 98 | 98 | 4900 | 1 901 592 |
| 64 | 122 | 122 | 7808 | 3 721 000 |
| 100 | 198 | 198 | 19800 | 15 603 192 |

### 5.2.2. Networks With $m$<2$n$–2 According to Section 3

As mentioned above the values of $r$ are rather small in this case ($r \leq n/2$). This has unfavourable consequences for the cross point requirements. In tabl. 3, some networks of this kind are listed for values of $m$=2$n$–1–$g$ ($g \geq 2$). For systems with $g$=2 and $r \leq n/2$ it can easily be shown that in all cases there exists a corresponding Clos network with fewer crosspoints. This is of course prohibitive for a realisation. Therefore these networks as listed in tabl. 3 (according to section 3) are only of theoretical interest.

## 6. Conclusion

In this paper it has been shown that, in addition to the well known example by Paull [4], there exist further networks which are rearrangeable with the reswitching of at most one connection. For systems with $m$=2$n$–2 middle switches this is possible if the number $r$ of input switches and output switches, respectively, has the maximum value 2$n$–2. Furthermore a new type of networks is presented with $m$=2$n$–1–$g$ middle switches ($g \geq 2$). These networks are rearrangeable by reswitching of at most one connection if the number $r$ of input switches and output switches, respectively, has the maximum value $r_{max}=(n-2)/g+1$. The results are illustrated by examples.

*Table 3*: Networks with $m \leq 2n$-3 according to section 3

| g | n | m | r | N | CP | matrix |
|---|---|---|---|---|---|---|
| 2 | 6 | 9 | 3 | 18 | 405 | 324 |
| 2 | 7 | 11 | 3 | 21 | 459 | 441 |
| 2 | 8 | 13 | 4 | 32 | 1 040 | 1 024 |
| 3 | 8 | 12 | 3 | 24 | 684 | 576 |
| 2 | 9 | 15 | 4 | 36 | 1 320 | 1 296 |
| 3 | 9 | 14 | 3 | 27 | 882 | 729 |
| 2 | 10 | 17 | 5 | 50 | 2 125 | 2 500 |
| 3 | 10 | 16 | 3 | 30 | 1 104 | 900 |
| 4 | 10 | 15 | 3 | 30 | 1 035 | 900 |
| 2 | 20 | 37 | 10 | 300 | 18 500 | 40 000 |
| 2 | 50 | 97 | 25 | 1 250 | 303 125 | 1 562 500 |
| 2 | 100 | 197 | 50 | 5 000 | 2 462 500 | 25 000 000 |

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