SOME PROBLEMS OF SOFT HANDOFF MODELING

A.S.Rodionov, V.S.Antyufeev[1], H.Choo, and H.Y.Youn[2]

[1] – Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch of the Russian Academy of Sciences, Lavrentjeva, 6, Novosibirsk, 630060, Russia, alrod@rav.sscc.ru, ant@osmf.sscc.ru

[2] – School of Electrical and Computer Engineering, Sungkyunkwan University, 440-746, Suwon, Korea, choo, youn@ece.skku.ac.kr

ABSTRACT

In this paper some models for obtaining the distribution of a sojourn time in CDMA cellular systems are proposed. Knowing this distribution is necessary for reliable modeling of the soft handoff and for solving some other problems in the cellular systems analysis. Proposed model is based on the random walks approach and can be adopted for different conditions. Analytical results can be obtained for simple and unfitted cases so simulation models are used for Monte Carlo experiments. Main assumptions include different kinds of mobile carriers (pedestrians and transport passengers) and round form of a cell. The scheme of simulation experiments is presented along with the discussion of simulation results.

1. INTRODUCTION

It is known, that the same frequency band can be used simultaneously over neighboring cells in CDMA cellular systems. This enables so called soft handoff scheme, when the new base station (BS) is assigned to a mobile while this mobile is still served by old one and will be served by it until reach some *outer handoff border*. The assign of new BS occurs when a mobile reaches some *inner handoff border* or just starts call inside *handoff zone*. In the last case two or more stations are assigned to this mobile, but for distinctness we will consider one of them as "old".

For different models of the soft handoff it is necessary to have distribution of the sojourn time or time when an active (busy by call) mobile is inside the handoff zone. Unfortunately, there is no real available data on the sojourn time, so some authors make attempts to obtain this distribution from some plausible (sometimes not very) reasoning. In last years some papers concerning this problem were published. Our attention was attracted by [1] and we have tried to improve the model proposed by its authors. The use of round form of a cell seems good choice for us but we can't to agree with the model of a mobile movement that was proposed in this paper. Than we found new results of the same research group [2]. In this paper authors tried to simplify the problem by considering the square form of a cell instead of the round one. We think this was a wrong decision as the real simplification of our problem can be made in a quite other direction. Proposed models are based on the random walks approach [3,4] and can be adopted for different conditions. Analytical results are obtained and simulation models are used for Monte Carlo experiments [5].

The rest of the paper is organized as follows. Section 2 is devoted to the main modeling assumptions, notations and concepts. In Section 3 we explain how to reduce the problem to one-dimensional one. Main distributions are discussed there also.. In Section 4 we discuss obtaining the probability of a mobile appearance on the border of the sojourn zone. Analytical model is discussed there in separate subsection. Section 5 presents the description of simulation model and simulation results. Section 6 is a brief conclusion.

2. MODELING ASSUMPTIONS

In order to simplify the modeling of the soft handoff we have the following assumptions:

- a base station is located in the center of polar coordinates;
- the aerial of BS is omni-directional;
- a cell has round form;
- soft handoff region has ring form and lays between inner and outer bounds that are defined by radiuses r and R respectively (see fig. 1);
- a mobile can move all directions and has a non-zero probability ν of been motionless;
- a mobile can start call at any point (x,y) of a cell uniformly (in Cortesian coordinates).
- observations of a mobile position are made with constant time intervals Δt, so time of i-th observation is $i\Delta t$;
- call durations Tc, are independently and equally distributed random variables. The distribution (let us denote its cumulative function as $G(t)$) is renewal one.

3. REDUCING TO ONE-DIMENSIONAL PROBLEM

Among the base assumptions mentioned above we will present some more that from one side require explanations and from other helps us greatly simplify the problem.

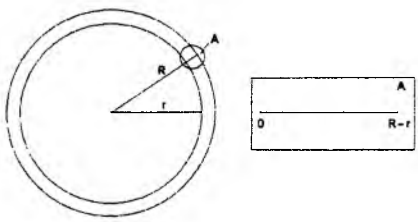

Figure 1. Sojourn area and its one-dimensional projection

First. As *only* the time in a handoff zone is of interest, we can just assume that a call starts with some probability ω on a zone bound, or uniformly at any point inside it. So we give no attention to mobiles inside the inner round. As no difference exists for sojourn between exiting the sojourn zone into an inner zone of any neighbor cell and motions are omni-directional, we can consider all incoming into the sojourn zone mobiles as penetrating through one border (say, inner one).

Second. As inter-observation intervals are constant, we can assume that a mobile velocity is step function unchanged on them. So distance that a mobile goes between observations at time t_i and t_{i+1} is simply $V_i\Delta t$. For more adequacies we also assume that there is non-zero probability ν that this distance is 0 (mobile was immovable or moved without changing a distance from the BS). Each time when the inter-observation interval comes to an end the new value of velocity is chosen accordingly to a given probability distribution. Values are independent and equally distributed.

Third. As any movements along the ring are not interesting from the point of sojourn time, we can analyze only there projection on the radial direction. So our task is one-dimensional one (!) (see fig. 1).

Note that in our models we need not the distribution of inter-calls intervals. We need not any information about number of mobiles in the sojourn area (or in cell) also.

4. Obtaining The Probability Distribution Of A Sojourn Time

Now we will discuss possible initial distributions and obtain corresponding distribution of a sojourn time.

As it is usual practice, we assume that random walk process is renewable. From this we have that mobiles that are detected on the inner border are treated uniformly as all of them are just started connection.

Let us define the distribution of speed in the radial direction. As the most plausible variant we choose the following.

$$F_V(x) = \begin{cases} (1-\nu)F(x), & \text{if } x<0; \\ \nu + (1-\nu)F(x), & \text{otherwise.} \end{cases} \quad (1)$$

Here $F(x)$ is some cumulative function of some distribution (its form is discussed later) with non-zero expectation V, and ν is the non-zero (in a common case) probability of zero speed as it was mentioned above.

The form of a function $F(x)$ we obtain from the following assumptions:

1. user of a mobile can be pedestrian or driver (passenger) of some transport;
2. user can change his status during the call time (pedestrian takes a seat in a bus or passenger goes out from a taxi, for example).

From this we have that

$$F(x)=\delta F_p(x)+(1-\delta)F_t(x) \quad (2)$$

where $F_p(x)$ and $F_t(x)$ are the cumulative distribution functions of velocity for cases of pedestrian and vehicle respectively, δ is the probability that the user is a pedestrian. So the final common expression for the cumulative distribution function of a velocity is

$$F_V(x) = \begin{cases} (1-p)[\delta F_p(x)+(1-\delta)F_t(x)], & \text{if } x<0; \\ p+(1-p)[\delta F_p(x)+(1-\delta)F_t(x)], & \text{otherwis} \end{cases} \quad (3)$$

As for the kind of $F_p(x)$ and $F_t(x)$ we choose normal (Gauss)distributions with expectations m_p and m_t, and standard deviations σ_p and σ_t correspondingly.

Now the task of obtaining the distribution of the life-time of an active mobile inside the sojourn zone T_s is reduced to obtaining the distribution of minimum value from pair $\{Tz_c, T_s\}$, where T_z is time of a mobile existence inside the sojourn zone and Tz_c is part of call duration from the moment when a mobile enters the sojourn zone to a call end. Using our assumption about the distribution of a call duration being renewal and from (3) we have:

$$F_s(x) = P(\min\{Tz_c, T_s\} < x) = P(Tz_c < x \vee T_s < x) =$$
$$\begin{cases} F_{Tz_c}(x) + F_{T_s}(x) - F_{Tz_c}(x)F_{T_s}(x), & \text{if } x > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

So our task is to obtain the distribution of T_s. To solve this problem analytically is not simple. The solution requires special methods that are explored in the theory of *random walks* (Brownian motion is a partial case). Those who is interested can refer, for example, to [5] and [6]. Here we present some analytical results along with experimental results obtained by the Monte Carlo method.

4.1. Distribution Of T_z

Thus we have the following problem. At the moment t=0 the random point is distributed over the interval (a,b) with a density $h(t)$. At the moments

t=1,2,... it makes motions of random length X_t (X_t can be of any direction). The lengths of all motions are distributed with an identical density $f(t)$ on a real axes (-∞, ∞). We need to find:

1. distribution of the moment of the first exit of a point from the interval (a,b);
2. a probability P_N that the point for the first time leaves the interval (a,b) at the moment $t=N$.

We denote by N the number of motions that is needed for the first leave from the interval (a,b). The distribution of N is

$$P(N=n)=p_n,\ p_0+p_1+\ldots=1.$$

First we find the distribution of N.

Denote by B_k the following event: the point for the first time leaves the interval at k-th step. Let the probability of event B_k be known (we will know how find it after solving the task 1): $P(B_k)=P_k$. The event B_k does not depend on event $\{N=k\}$. Therefore the probability of occurrence of event $A_k=\{N=k\}B_k$ is equal to the product p_kP_k. The events A_k are incompatible at different values k, therefore the probability P that the point for the first time leaves the interval (a,b) at the random moment N is equal to the sum $p_1+p_2+\ldots$. Thus we need to find the probabilities P_k, that is to solve the task 1.

Consider in detail how to obtain the density of the points, which have not left the interval (a,b) at the moment t=1, that is after the first motion. The distribution of such random point coincides with distribution of the random variable $Y+X_1$. The density of this sum is equal to convolution $[h*f]$. Let:

$$p_0=\int_a^b h(t)dt,\ h_0(t)=h(t)$$

(certainly, p_0=1, as h is normalized density); $\chi_{(0,1)}$ – characteristic function of interval (a,b), i.e.

$$\chi_{(a,b)}(t)=\begin{cases}1, & t\in(a,b),\\ 0, & t\notin(a,b).\end{cases}$$

So density of a random variable $Y+X_1$ is equal to

$$[h_0*f](x)=\int_a^b h_0(t)f(x-t)dt$$

It is a density function of a point after the first motion. No limitations are imposed on a position of the point, after a motion the point can leave the limits of the interval (a,b). Normalized distribution density h_1 of the point after the first motion provided that it has not left the interval (a,b), is equal to

$$h_1(x)=\frac{1}{p_0}\chi_{(a,b)}(x)[h_0*f](x) \qquad (5)$$

So, if at the moment t=0 the normalized density function of the point in the interval (a,b) is h_0, then it is possible to obtain normalized distribution density h_1 of a point in the interval (a,b) at the moment t=1 by the formula (5).

Let h_k be the normalized density function of points in the interval (a,b), which in the first k motions have never left the interval (a,b). Then, in the same way as for transition $h_0\to h_1$ from the density h_0 to the density h_1, we obtain the recursive formula for transition $h_k\to h_{k-1}$:

$$h_{k+1}(x)=\frac{1}{p_k}\cdot\chi_{(a,b)}(x)\cdot[h_k*f](x) \qquad (6)$$

where

$$p_k=\int_a^b[h_k*f](t)dt.$$

The value p_k is a probability that the point which has not left the interval (a,b) in k-1 motions, will not leave it on k-th step. Therefore, the probability P_k that the point will leave the interval (a,b) at k-th step for the first time, is equal to 1-p_k.

Thus we have the following algorithm to obtain the values P_k. Successively, we obtain densities $h_0\to h_1\to h_2\to\ldots$ by the recursive formula (6). After we find the next density $h_k(t)$, we obtain probability P_k by the formula

$$P_k=1-\int_a^b[h_k*f](t)dt.$$

Let's make the short remark on computational algorithm for the density h_k and probabilities P_k. We can change the boundaries of the interval (a,b) by an appropriate linear change of variables. Transform the interval (a,b) *to* (-1,1). Rewrite in more detail the formula (6):

$$h_{k+1}(x)=\frac{1}{p_k}\cdot\int_{-1}^1 h_k(t)f(x-t)dt \qquad (7)$$

Note, that to calculate this convolution we only need to know the values of the function f on the interval (-2,2), and the function h is defined on an interval (-1,1). Expand the functions h and f to Fourier series on appropriate intervals. Using the coefficients of expansion, we can obtain simple recursive expressions for these coefficients for series of functions h_k and integrals P_k.

4.2. Initial Data

Here we try to choose reliable or at least reasonable initial data for simulation.

First to all let us to estimate parameters of initial distributions. Average value of a pedestrian speed lies obviously somewhere between 3 and 6 km while average value of a speed of some vehicle ought to be chosen from wider range and highly depends on area: it may be from 20 to 40 km for city and up to 70-80 km for a country. We consider the projection of a moving vector on a cell radius so the average absolute value of speed with which carrier goes to the border is $V_0=2V/\pi$, where V is linear speed of a carrier. The multiplier is simply obtained by transformation of a motion in Cartesian rectangular coordinates into motion in polar ones. So the average speed with which a pedestrian goes to a cell border is about 2-4 km while these speed for a vehicle is about 22-27 km for a city and 47-53 km for a country region. As for a reliable variance, we obtain it from a simple reason: the probability of a speed being more than twice from average ought to be negligible, say not more than 1%. So, if an average speed of a pedestrian is 3 km then correspondent standard deviation is 1.29 km.

Next we need the probability ω of an active mobile entering the sojourn zone through border (call does not start inside the zone but continues). For this we use the following simulation: calls starts inside the inner part of a cell (radius r), motions are simulated according to the scheme described above and M_e – a number of mobiles leaving the inner part while call have not yet ended, is stored. Let M be the whole number of simulated calls. Note that part of M in the whole number of calls started inside the cell (radius R) is r^2/R^2. So ω is obtained as:

$$\omega=M_e r^2/[M_e r^2+M(R^2-r^2)].$$

For cells large enough we can consider the flows of mobiles through inner and outer borders as equal and so we can simplify our simulation model and treat all the incoming calls as entering through inner border. Then

$$\omega=2M_e r^2/[2M_e r^2+M(R^2-r^2)].$$

The division of carriers into pedestrians and passengers is not constant and greatly depends on area: almost all are pedestrians inside a city park and almost all are passengers on the highway crossing. So we have tried δ=0.0,0.1,0.2…,1.0.

The intensity of a call flow is of no difference for our task. The average call duration $\overline{T_c}$ we choose to be equal to 3 minutes that is rather common choice. But we made experiments with $\overline{T_c}$ equal to 2 and 4 minutes also.

We need also the value of ν – the probability of a mobile being immovable. This probability can significantly differ in different areas: on a road it is obviously smaller than in a city park or recreation zone. We have no real reliable information about its value but been guided by common sense we simply made simulation with different $\nu\in[0,0.3]$.

As for cell radius and width of sojourn zone we choose R equal to 400,500,800,1200 and 2500 m and r equal to 0.8 or 0.9 of R.

Table 1. The values of ω obtained by simulation

δ	0.2	0.5	0.8
R=400m	0.684	0.677	0.652
R=500m	0.682	0.673	0.644
R=800m	0.673	0.660	0.618
R=1200m	0.662	0.644	0.583
R=2500m	0.626	0.589	0.489

5. SIMULATION RESULTS

The simulation model is really rather simple and need not any additional explanation. Here the most interesting results are presented. Time interval Δt between changes of speed is equal to 5 sec.

5.1. Dependency Of ω From A Cell Radius

Making 100000 experiments for each variant with $r=0.8R$ and $\nu=0.1$ we obtain for $\overline{T_c}=3$ min the results presented in tabl. 1. Simulation takes about 10 seconds for value in average on PC with Intel Pentium II, 400 Mz inside.

It is easy to see that value of ω decreases with the growth of a cell radius. This is due to the fact that with the growth of a cell radius more calls have time enough for completion inside the inner part of a cell.

5.2. Distribution Of A Sojourn Time

Simulation had been made for some combinations of parameters. Values of ω were taken from the table presented in the previous subsection. As it was expected, experiments show that the distribution of a sojourn time greatly depends on the zone width and average speed of a mobile carrier; the last depends on division of carriers on pedestrians and passengers. In fig. 2 and fig. 3 we can see this clearly. In first case average sojourn time is about 8 seconds (maximum – 45 sec) while in second, where part of pedestrians is a majority, average sojourn time is about 17 seconds (maximum – 135 sec).

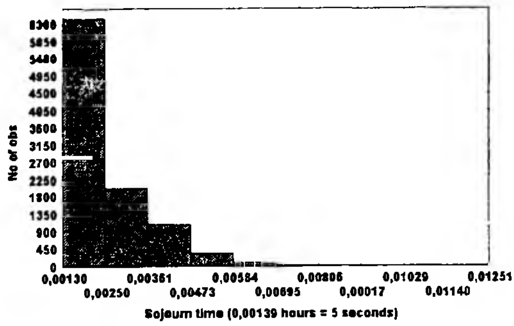

Figure 2. Histogram of sojourn time for R=400m, r=320m, δ=0.2

For large cells the dependency on average speed is even more. In tabl. 2 the results of experiments with a cell outer radius equal to 2500 m and different inner radiuses are presented. The number of experiments for each radius was 100000, v=0.05, δ=0.2.

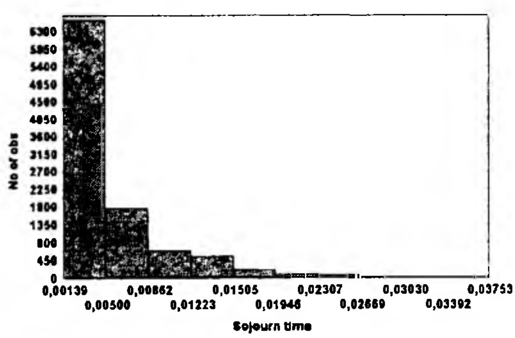

Figure 3. Histogram of sojourn time for R=400m, r=320m, δ=0.8

In fig. 4 the dependency of an average time of sojourn from the width of a zone is presented (column 3 of the tabl. 2). It is clear that dependency is almost linear. The same is with maximum value also.

Table 2. Dependency of sojourn parameters from the width of sojourn zone (times are in minutes)

r	ω	Average T_s	Maximum T_s
2000	0.626	0.00682	0.0278
2050	0.669	0.00628	0.0264
2100	0.712	0.00577	0.0250
2150	0.752	0.00524	0.0236
2200	0.792	0.00471	0.0236
2250	0.831	0.00413	0.0209
2300	0.868	0.00355	0.0181
2350	0.904	0.00299	0.0167
2400	0.937	0.00242	0.0139
2450	0.970	0.00176	0.0125

Detailed simulation of the motions of an active mobile inside the sojourn zone, made with assumption of a round form of a cell gives results that differs from results presented in [1,2]. It seems to us that our assumptions are more reliable than those in these papers but the final choice can be made only on analysis of real data that is inaccessible for us.

6. Conclusions

Thus, using simulation technique we have obtained some results on analysis of distribution of the sojourn time in TDMA systems. Analytical approach to calculation of main parameters is also presented. We believe that our present work is of some help to those who makes researches in this area.

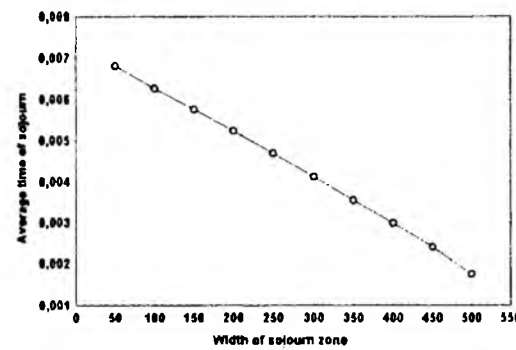

Figure 4. Dependency of an average time of sojourn from the width of a zone

References

[1]. Suwon Park, Ho Shin Cho, and Dan Keun Sung, "Modeling and Analysis of CDMA Soft Handoff", *VTC'96,* Atlanta, U.S.A., pp. 1525-1529, 1996.

[2]. Jae Kyun Kwon and Dan Keun Sung, "Soft Hadoff Modeling in CDMA Cellular Systems," *Vehicular Technology Conference,* IEEE 47th, Vol. 3, pp. 1548-1551, 1997.

[3]. W.Feller, *An introduction to probability theory and its applications,* John Wiley\&Sons, 1971.

[4]. F.Spizer, *Principles of random walk,* Princeton, New Jersey, 1964.

[5]. J.M.Hammersley and D.C.Handscomb, *Monte-Carlo Methods*, Methuen&Co, Ltd, 1964.

[6]. T.~Hida, *Brownian Motion*, Springer-Verlag, 1980.