 

# DYNAMIC DETERMINISTIC RELIABILITY INDICES

Elena N. Zaitseva

Department of Information Technologies, Belarus State Economic University, Partizan ave., 26, 220672, Minsk, BELARUS, zaitseva_e@yahoo.com

## ABSTRACT

The Multi-State System (MSS) reliability is investigated in this paper. It is a system, which can have some different task performance levels. The original investigation direction of reliability analysis for this system by using Multiple-Valued Logic and the Direct Partial Logic Derivatives develops in this paper. The new class of Dynamic Reliability Indices is obtained. These indices estimate influence upon the Multi-State System reliability by the capacity levels of a system component.

## 1. INTRODUCTION

In this paper the *Multiple-Valued Logic* (MVL) is applied for MSS reliability analysis [5]. In this case a structure function is interpreted as a MVL function. MVL approach causes to synthesize special reliability indices [5, 6]. We have used the Direct Partial Logic Derivatives of MVL functions for calculation of reliability indices. This approach is permitted to investigate influence upon the MSS reliability by the capacity levels of a system component [7].

Firstly the Direct Partial Logic Derivatives of MVL functions for MSS reliability analysis had proposed in [6, 7]. The new class of *Dynamic Reliability Indices* (DRI) had obtained in these papers. The authors had defined the *Dynamic Deterministic Reliability Indices* (DDRI) and had synthesised the algorithm for they calculation. We have continued these investigations and have proposed new indices of the DRI class. These indices determine a probability evaluation of a MSS reliability modification when a component changes capacity levels.

## 2. PROBLEM FORMULATION AND DESCRIPTION OF SYSTEM MODEL

A system consisting of $n$ components is considered. The system and its components have $m$ states (level to performance a task or serviceability): from complete failure up to perfect functioning. The failure is corresponds to zero state. The best state is ($m$-1). The state of system is determined by the states of its components.

So, the component states are denoted as [5]:

$$x_i(t)=\mathrm{W}_i \qquad (1)$$

where $\mathrm{W}_i \in \{0, 1,\ldots,m\text{-}1\}$ is numerical characteristic for the state of $i$-th component at instant $t$; $x_i(t)=0$ corresponds to the failure and $x_i(t)=m\text{-}1$ corresponds to the best state.

In this paper a system is analyzed in fixed instants. In [5, 9, 10] is shown that for this system instead of functions $x_i(t)$ we can use variables $x_i \in \{0, 1,\ldots,m\text{-}1\}$.

The MSS behavior is characterized by its evolution in the space of states:

$$f(x_1, x_2, \ldots, x_n)\colon \{0, 1,\ldots,m\text{-}1\}^n \to \{0, 1,\ldots,m\text{-}1\}, \qquad (2)$$

where $f(x_1, x_2,\ldots,x_n)=f(\mathrm{X})$; $f(\mathrm{X})=0$ corresponds to the system failure and $f(\mathrm{X})=m\text{-}1$ corresponds to the best state of it.

The relation (2) is MVL function, which is interpreted as structural function in a reliability theory. MVL function will be represented as truth table $\mathrm{X}=[x^{(0)}\ x^{(1)},\ldots,x^{(m^n\text{-}1)}]$. The problem of structural function synthesis is complex, because dimension of this is $m^n$ ($m$ and $n$ are numbers of system states and system components accordingly). Therefore decomposition of a system is necessary for reliability analysis.

For example, the structural function for the system in fig.1 is shown in tabl.1. This system and its components have three states: $m \in \{0, 1, 2\}$.

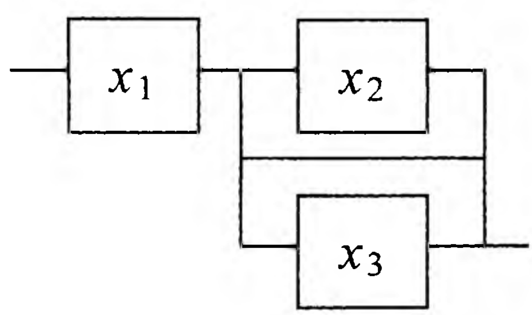


*Figure 1.* Example of MSS.

## 3. Reliability Indices of MSS

There are three classes of indices of system reliability according to publications in the reliability theory (fig.2).

*Table 1*: The structural function of MSS ($m$=3).

| $x_1\,x_2\,x_3$ | $f$(X) | $x_1\,x_2\,x_3$ | $f$(X) | $x_1\,x_2\,x_3$ | $f$(X) |
|---|---|---|---|---|---|
| 0 0 0 | 0 | 1 0 0 | 0 | 2 0 0 | 1 |
| 0 0 1 | 0 | 1 0 1 | 1 | 2 0 1 | 1 |
| 0 0 2 | 0 | 1 0 2 | 1 | 2 0 2 | 1 |
| 0 1 0 | 0 | 1 1 0 | 1 | 2 1 0 | 2 |
| 0 1 1 | 0 | 1 1 1 | 2 | 2 1 1 | 2 |
| 0 1 2 | 1 | 1 1 2 | 2 | 2 1 2 | 2 |
| 0 2 0 | 0 | 1 2 0 | 2 | 2 2 0 | 2 |
| 0 2 1 | 1 | 1 2 1 | 2 | 2 2 1 | 2 |
| 0 2 2 | 1 | 1 2 2 | 2 | 2 2 2 | 2 |

$x_1$ is the 1-st component; $x_2$ is the 2-nd component; $x_3$ is the 3-rd component.

*The first class* is deterministic reliability indices for analysis MSS structure. As a rule the deterministic indices are defined by the mathematical approach of the graph theory [5, 10]. These indices are used in a step of the system design.

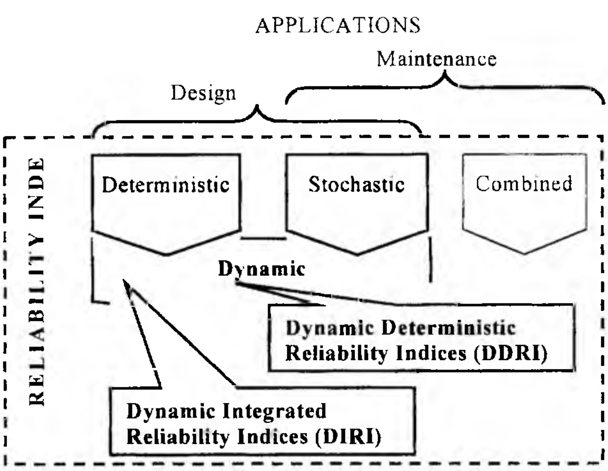


*Figure 2*: The classification of reliability indices.

*The second class* of indices is stochastic indices. They allow estimating a reliability of a system without the analysis of exterior influences. The mathematical statistic and probability theory use to form these indices. They are applied for an evaluation of system reliability during its work [9, 11, 12].

*The third class* of reliability indices includes indices, which characterize operation of a system with exterior influence [2].

In this paper we investigate the DRI. These indices allow to consider sensitivity of a system reliability to a change of a level of quality of operation a component. The DRI consist of two indices types.

The first DRI type is DDRI. They permit to analyze MSS reliability on a design stage (fig.2). DDRI were defined in [6]. The calculation algorithm by the mathematics app-roach of Logic Differential Calculus was proposed in [8].

The *Dynamic Integrated Reliability Indices* (DIRI) are the second DRI type. In this paper the DIRI are considered firstly. These indices allow to define a probability evaluation of a MSS failure or repair for modification of a level of serviceability of a system component.

Form the computational point of view, the calculation of DRI is sensitivity investigation of structural function which is given as MVL function.

Mathematical tools of Logic Differential Calculus calculate the sensitivity of MVL function [7]. We have considered DRI calculation technology below. The Logic Partial Derivatives of MVL function, which are part of Logic Differential Calculus, were used in this technology.

## 4. The Dynamic Reliability Indices

### 4.1. The Dynamic Deterministic Reliability Indices

The DDRI evaluate an influence of a level change of a component state on system reliability. Numerical DDRI is defined as sets of the system states. These sets are calculated by Direct Partial Logic Derivatives [7].

***Definition 1*** DDRI are sets of a MSS states {G} which modify level to performance a task from $a$ to $b$ if a component serviceability level (component state) changes from $t$ to $k$. The states system {G} is union of subsets $\{G|x_i\}$. The subset elements are system states for which a change of a serviceability level of $i$-th component from $t$ to $k$ causes a change of the system level to performance a task from a to b.

The mathematical interpretations of DDIR are

$$\{G\}=\{G|x_1\}\cup\{G|x_2\}\cup\ldots\cup\{G|x_n\}, \quad (3)$$

where subsets elements $\{G|x_i\}$ correspond the values $t_1\ldots t_{i-1}t_{i+1}\ldots t_n$ of variables $x_1\ldots x_{i-1}x_{i+1}\ldots x_n$ for which the values of Direct Partial Logic Derivative $\partial X(j\to l)/\partial x_i(a\to b)$ with respect to $i$-th variable are not equal 0 (X is the truth table of the structure function (2) ):

$$\partial X(j \to l)/\partial x_i(a \to b) \neq 0. \quad (4)$$

The four reliability indices from DDRI are defined in tabl. 2. These Direct Partial Logic Derivatives for calculation they are shown in this table too.

### 4.2. The Dynamic Integrated Reliability Indices

The DIRI is generalization of the DDRI. The DIRI is a probable evaluation of a modification of system boundary states depending on a serviceability level change of a system component.

***Definition 2.*** DIRI are probability of a modification of level to performance a task from $j$ to $l$ if a component serviceability level (component state) changes from $a$ to $b$.

$$g = \sum_{i=1}^{n} P(i)_{a \to b}^{j \to l}, \quad (5)$$

where we have compatible events; $P(i)_{a \to b}^{j \to l}$ is probability of a modification of system level to performance a task from $j$ to $l$ if $i$-th component serviceability level changes from $a$ to $b$: $P(i)_{a \to b}^{j \to l} = \rho(i)_{a \to b}^{j \to l} / m^n$; $\rho(i)_{a \to b}^{j \to l}$ is number of system states for which a change $i$-th component serviceability level changes from $a$ to $b$ causes a modification of system level to performance a task from $j$ to $l$.

The $\rho(i)_{a \to b}^{j \to l}$ is number of values of Direct Partial Logic Derivative $\partial X(j \to l)/\partial x_i(a \to b)$ with respect to $i$-th variable (4) which are not equal 0.

In tabl. 2 are defined some DIRI.

## 5. Calculations of the Dynamic Reliability Indices

Let's consider an example of a Dynamic Reliability Indices calculation of a MSS (see Fig.1). Its structure function is given in Table 1. We used Direct Partial Logic Derivatives of MVL functions for a calculation. Dynamic Reliability Indices of this system is shown in tabl. 3.

The analysis of the indices from tabl. 3 allows to make the following comments:

1. The probability of a system failure with a component failure is higher (0.19) than for decrease a serviceability level one of a system component (0.06).
2. So, the system is coming to failure, if:

a) the 1-st component is coming to failure and
- the 2-nd component is failure too;
- the 2-nd component has state 1 and the 3-rd component is failure;

b) the 2-nd component is coming to failure and
- the 1-st component is failure and the 3-rd component has state 2;
- the 1-st component has state 1 and the 3-rd component is failure;

c) the 3-rd component is coming to failure and
- the 1-st component is failure and the 2-nd component has state 2;
- the 1-st component has state 1 and the 2-nd component is failure.

3. The MSS is failure, if
- the 1-st component is reduce states and the 2-nd and 3-rd component are failures;
- the 3-rd component is reduce states and the 1-st component is failure and the 2-nd component has state 2.

4. The probability of a system repairing above by replacement of the failure component (0.19) than for increases a serviceability level of a system component (0.06).
5. The system failure can be eliminated by the replacement of:
- the 1-st component if the 2-nd component is failure (in this case the state of the 3-rd component is not important);
- the 2-nd component if the 1-st component is failure and the 3-rd component is not failure;
- the 3-rd component if the 1-st component is failure and the 2-nd component is not failure.

6. For eliminating the system failure is necessary to increase the states of:
- the 1-st component if 2-nd and the 3-rd components are failures;
- the 3-rd component if the 1-st component is failure and the 2-nd component has state 1.

We would like to mark that the first and the fourth comments are based on DIRI, and remaining on DDRI. Therefore, DIRI allow to give an initial common evaluation of boundary condition of a system. There is a possibility precisely to determine boundary condition of a system with usage of DDRI.

## 6. Conclusions

The new reliability indices for MSS were offered in the paper. It is dynamic reliability indices. The mathematical tool of MVL and Logic Differential Calculus is used for calculation of these indices. In particular we offer to use Direct Partial Logic Derivatives for calculation of dynamic reliability indices.

These indices allow to investigate an outcome of a modification of a serviceability level of separate a system components on a condition of a system in a reliability theory. These indices can be determined on a design stage of a system.

The authors' further investigations consist in using of these indices for optimization of MSS reliability.

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## APPENDIX. SHORT INFORMATION ABOUT LOGIC DIFFERENTIAL CALCULUS OF MVL FUNCTIONS

***Definition A.1.*** A Direct Partial Logic Derivative $\partial X(j\to l)/\partial x_i(a\to b)$ of a $m$-valued logic function X of $n$ variables with respect to variable $x_i$ reflects the fact of changing of function from $j$ to $l$ when the value of variable $x_i$ is changing from $a$ to $b$ and it is computed as

$$\partial X(j\to l)/\partial x_i(a\to b)=(P_{m^n}^{(i,a)}\cdot\varphi_j(X))\cdot(P_{m^n}^{(i,b)}\ \varphi_l(X)),\quad (A)$$

where vector literal $\varphi_s(X)$ ($s=j,\ l$) represented as

$$\varphi_s(X)=\varphi_s([x^{(0)}\ldots x^{(m^n-1)}])=[\varphi_s(x^{(0)})\ldots\varphi_s(x^{(m^n-1)})].$$

$$\varphi_s(y)=y^s=\begin{cases}0, & y\neq s\\ m-1, & y=s.\end{cases}$$

The matrix $P_{m^n}^{(i,k)}$ ($k=a,\ b$) is formed by the rule

$$P_{m^n}^{(i,k)}=M_m i\text{-}1\otimes P_m^{(k)}\otimes M_m n\text{-}i,\qquad (B)$$

where $\otimes$ denotes Kronecker product; $M_m i$-1 and $M_m n$-$i$ are the $m^{i-1}\times m^{i-1}$ and $m^{n-i}\times m^{n-i}$ diagonal matrices containing the value ($m$-1) in main diagonal; the matrix $P_m^{(k)}$'s structure is the

$$P_m^{(k)}=[\varphi_k(0)\ \varphi_k(1)\ldots\varphi_k(m\text{-}1)]\otimes\begin{bmatrix}m-1\\ m-1\\ \vdots\\ m-1\end{bmatrix}\qquad (C)$$

*Example A.1.* Form the matrices $P_{3^2}^{(1,1)}$ and $P_{3^2}^{(1,2)}$ using (B) and (C):

$$P_{3^2}^{(1,1)}=P_3^{(1)}\otimes M_3=\begin{bmatrix}0&2&0\\0&2&0\\0&2&0\end{bmatrix}\otimes\begin{bmatrix}2&&\\&2&\\&&2\end{bmatrix}=\left[\begin{array}{ccc|ccc|ccc}&&&2&&&&&\\&&&&2&&&&\\&&&&&2&&&\\ \hline &&&2&&&&&\\&&&&2&&&&\\&&&&&2&&&\\ \hline &&&2&&&&&\\&&&&2&&&&\\&&&&&2&&&\end{array}\right].$$

$$P_{3^2}^{(1,2)}=P_3^{(2)}\otimes M_3=\left[\begin{array}{ccc|ccc|ccc} & & & & & & 2 & & \\ & & & & & & & 2 & \\ & & & & & & & & 2 \\ \hline & & & & & & 2 & & \\ & & & & & & & 2 & \\ & & & & & & & & 2 \\ \hline & & & & & & 2 & & \\ & & & & & & & 2 & \\ & & & & & & & & 2 \end{array}\right].$$

*Example A.2.* Compute the Direct Partial Logic Derivative $\partial X(0\to 1)/\partial x_1(1\to 2)$ of the 3-valued logic function $f(X)$ of 2 variables given by its truth table $X=[111200210]^T$.

Use expression (A) and write

$\partial X(0\to 1)/\partial x_1(1\to 2)=(P_{3^2}^{(1,1)}\cdot\varphi_0(X))\cdot(P_{3^2}^{(1,2)}\cdot\varphi_0(X))$,

where $\varphi_0(X)=[000022002]^T$, $\varphi_1(X)=[222000020]^T$; matrices $P_{3^2}^{(1,1)}$, $P_{3^2}^{(1,2)}$ are formed as in *Example A.1*. The computed vector $\partial X(0\to 1)/\partial x_1(1\to 2)=$ $=[020020020]^T$ corresponds to the following symbolic notation: $\partial f(0\to 1)/\partial x_1(1\to 2)=\overset{1}{x_2}$.

For instance, this notation describes the function, changing the state on the 1-st input from 1 to 2 results in changing the output from 0 to 1 when state on 2-nd input is 1.

The flow graphs of (A) when $m$=3; $n$=2; $a$=1, $b$=2; $i$=1,2 are shown in fig.A1. It should be noted that the structure of synthesized flow graphs coincides with the fast Fourier transform signal flow graphs structure used in digital signal processing.

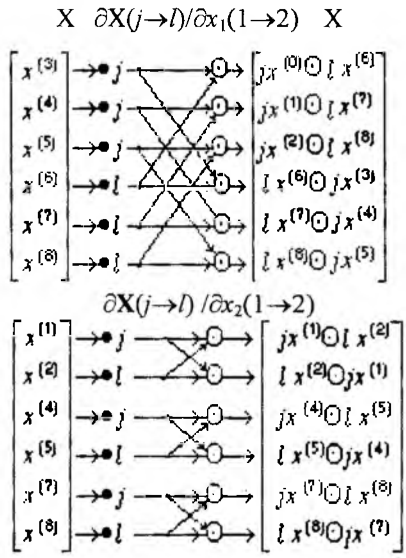


*Figure A1.* Flow graphs of the algorithms of computing the Direct Partial Logic Derivatives of MVL-function ($n$=2, $m$=3), symbol "•" is comparison operation $\varphi_s(y)$.

*Table 2.* The Dynamic Reliability Indices

| Reliability Indices | | Failure of system | | Repair of system | |
|---|---|---|---|---|---|
| | | *BRIF* | *TRIF* | *BRIR* | *TRIR* |
| **DDRI** | $i$=1 | $\{G_f\|_{x_1}\}=\{101, 102, 110\}$ | $\{\bar{G}_f\|_{x_1}\}=\{200\}$ | $\{G_r\|_{x_1}\}=\{000, 001, 002\}$ | $\{\bar{G}_r\|_{x_1}\}=\{100\}$ |
| | $i$=2 | $\{G_f\|_{x_2}\}=\{012, 110\}$ | $\{\bar{G}_f\|_{x_2}\}=\varnothing$ | $\{G_r\|_{x_2}\}=\{001, 002\}$ | $\{\bar{G}_r\|_{x_2}\}=\varnothing$ |
| | $i$=3 | $\{G_f\|_{x_3}\}=\{021, 101\}$ | $\{\bar{G}_f\|_{x_3}\}=\{012\}$ | $\{G_r\|_{x_3}\}=\{010, 020\}$ | $\{\bar{G}_r\|_{x_3}\}=\{011\}$ |
| | | $\{G_f\}=\{101, 102, 110, 012, 021\}$ | $\{\bar{G}_f\}=\{200, 012\}$ | $\{G_r\}=\{000, 001, 002, 010, 020\}$ | $\{\bar{G}_r\}=\{100, 011\}$ |
| **DIRI** | $i$=1 | $P(1)_{1\to 0}^{1\to 0}=0.11$ | $P(1)_{2\to 1}^{1\to 0}=0.03$ | $P(1)_{0\to 2}^{0\to 1}=0.11$ | $P(1)_{1\to 2}^{0\to 1}=0.03$ |
| | $i$=2 | $P(2)_{1\to 0}^{1\to 0}=0.07$ | $P(2)_{2\to 1}^{1\to 0}=0$ | $P(2)_{0\to 2}^{0\to 1}=0.07$ | $P(2)_{1\to 2}^{0\to 1}=0$ |
| | $i$=3 | $P(3)_{1\to 0}^{1\to 0}=0.07$ | $P(3)_{2\to 1}^{1\to 0}=0.03$ | $P(3)_{0\to 2}^{0\to 1}=0.07$ | $P(3)_{1\to 2}^{0\to 1}=0.03$ |
| | | $g_f=0.19$ | $\bar{g}_f=0.06$ | $g_r=0.19$ | $\bar{g}_r=0.06$ |
| | | $\frac{\partial X(1\to 0)}{\partial x_i(1\to 0)}$ | $\frac{\partial X(1\to 0)}{\partial x_i(2\to 1)}$ | $\frac{\partial X(0\to 1)}{\partial x_i(0\to 2)}$ | $\frac{\partial X(0\to 1)}{\partial x_i(1\to 2)}$ |

*Table 3.* The Dynamic Reliability Indices.

| | Dynamic Deterministic Reliability Indices* | Dynamic Integrated Reliability Indices* | The Direct Partial Derivatives |
|---|---|---|---|
| Failure of system | *The bottom reliability indices of failure (BRIF):* | | |
| | The set $\{G_f\}$ of the system states, for which the failure of one component causes system failure | The $g_f$ is probability of system failure if one of system components fails. | $\frac{\partial X(1\to 0)}{\partial x_i(1\to 0)}$ |
| | *The top reliability indices of failure (TRIF):* | | |
| | The set $\{\bar{G}_f\}$ of the system states, for which the reduction a serviceability level of a system component causes system failure. | The $g_r$ is probability of system failure if one of system components reduces a serviceability level. | $\frac{\partial X(1\to 0)}{\partial x_i(s\to s-1)}$ $(s = 2,\ldots,m-1$ |
| Repair of system | *The bottom reliability indices to repair (BRIR):* | | |
| | The set $\{G_r\}$ of the system failure states, which are eliminated by the replacement of a failure component. | The $\bar{g}_r$ is probability of system repair if one of failure system components is replaced. | $\frac{\partial X(0\to 1)}{\partial x_i(0\to m-1)}$ |
| | *The top reliability indices to repair (TRIR):* | | |
| | The set $\{\bar{G}_r\}$ of the system failure states, which are eliminated by the increase a serviceability level of a system component that is no failure. | The $\bar{G}_f$ is probability of system repair if one of system compo-nents increases a serviceability level that is no failure. | $\frac{\partial X(0\to 1)}{\partial x_i(s-1\to s)}$ $(s=2,\ldots,m-1)$ |

*We have used in these definitions next assumption: the structural function (2) is monotonous and it changes step by step. These assumptions are caused by conditions of MSS operation