RELIABILITY ANALYSIS AND OPTIMIZATION OF MULTI-STATE ACYCLIC DATA TRANSMISSION NETWORKS.

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ABSTRACT

The acyclic transmission networks (ATNs) consist of a number of positions (nodes) in which multi-state elements (MEs) capable of receiving and sending a signal are allocated. Each network has a root position where the signal source is located, a number of leaf positions that can only receive a signal, and a number of intermediate positions containing MEs capable of transmitting the received signal to some other nodes. The networks are arranged in such a way that no signal leaving a node can return to this node through any sequence of nodes (no cycles exist). Each ME that is located in a nonleaf node can have different states determined by a set of nodes receiving the signal directly from this ME. The probability of each state is assumed to be known for each ME. The ATN reliability is defined as the probability that a signal from the root node is transmitted to each leaf node.

In this paper, an algorithm for ATN reliability evaluation is suggested. The algorithm is based on using a universal generating function (UGF) technique.

It is shown that the proper distribution of MEs with different characteristics among ATN positions provides the ATN reliability enhancement. An algorithm for optimal allocation of multi-state elements in acyclic transmission networks is presented. A genetic algorithm (GA) is used as the optimization tool.

1. NOMENCLATURE

R ATN reliability
N total number of nodes in ATN
M number of leaf nodes in ATN
D number of MEs to be allocated at ATN
c_i i-th node of ATN
Δ set of MEs
Δ_i set of MEs allocated at c_i
λ_{ik} set of nodes receiving a signal from ME located at c_i when it is in state k
K_i number of different states of individual ME located at node c_i
$\tilde{K}_i$ number of different states of group of MEs located at node c_i
$\hat{K}_i$ number of different states of group of MEs located at nodes c_1, c_i
$p^d_{i\lambda_{ik}}$ probability that a signal from d-th ME located at c_i reaches set of nodes λ_{ik}
P^d state probability distribution matrix for ME d
V_i random binary vector representing set of ATN nodes receiving a signal directly from single ME located at node c_i
$\tilde{\mathbf{V}}_i$ random binary vector representing set of ATN nodes receiving a signal directly from group of MEs located at node c_i
$\hat{\mathbf{V}}_i$ random binary vector representing set of ATN nodes receiving a signal from c_1 through all the MEs located at $c_1,\dots,c_i$
V_{ik} value of V_i at state k (vector representing the set λ_{ik})
$\tilde{\mathbf{V}}_{ik}$ value of $\tilde{\mathbf{V}}_i$ at state k
$\hat{\mathbf{V}}_{ik}$ value of $\hat{\mathbf{V}}_i$ at state k
$\tilde{q}_{ik}$ probability that $\tilde{\mathbf{V}}_i$ is equal to $\tilde{\mathbf{V}}_{ik}$
$\hat{q}_{ik}$ probability that $\hat{\mathbf{V}}_i$ is equal to $\hat{\mathbf{V}}_{ik}$
$u_{id}(z)$ u-function corresponding to ME d located at node c_i (represents probabilistic distribution of V_i)
$\tilde{U}_i(z)$ u-function corresponding to group of MEs located at node c_i (represents probabilistic distribution of $\tilde{\mathbf{V}}_i$)
$\hat{U}_i(z)$ u-function corresponding to group of MEs located at nodes $c_1,\dots,c_i$ (represents probabilistic distribution of $\hat{\mathbf{V}}_i$)
φ u-function simplification operator
Ω,Ψ composition operators over u-functions
ω function for vector composition
$h(d)$ number of node in which ME d is allocated

2. INTRODUCTION

Acyclic transmission networks consist of a certain number of positions (nodes) in which multistate elements capable of receiving and/or sending a signal are allocated. Each network has a root node where the signal source is located, a number of leaf nodes that can only receive a signal and a number of intermediate (neither root nor leaf) nodes containing MEs capable of transmitting the received signal to some other nodes. The signal transmission is possible only along links between the nodes. The networks are arranged in such a way that no signal leaving a node can return to this node through any sequence of nodes (no cycles exist).

Each ME located in nonleaf node can have different states determined by a set of nodes receiving the signal directly from this ME. The event that a ME is in a specific state is a random event. The probability of this event is assumed to be known for each ME and for every its possible state. All the MEs in the network are assumed to be statistically independent.

The whole network is in working condition if a signal from the root node is transmitted to each leaf node. Otherwise, the network fails. (Note that it is not always necessary for a signal to reach all the network nodes in order to provide its propagation to the leaf ones).

An example of the ATN is a set of radio relay stations with a transmitter allocated at root node and a receivers allocated at leaf nodes. Each station has retransmitters generating signals that can reach a set of next stations. Note that the composition of this set for each station depends on power and availability of retransmitter amplifiers as well as on signal propagation conditions.

The acyclic transmission network is a generalization of the tree-structured multi-state systems investigated by Malinowski and Preuss [1] and multi-state linear consecutively-connected networks introduced by Hwang & Yao [2] and studied by Kossow & Preuss [3] and Zuo & Liang [4]. An algorithm for ATN reliability evaluation was suggested by Levitin in [5].

The problem of optimal ME allocation was first formulated by Malinowski & Preuss in [6] for linear consecutively-connected system. In this problem, MEs with different characteristics should be allocated in system nodes in such a way that maximizes the system reliability. A multi-start local search algorithm was suggested for solving this problem.

This paper presents an algorithm for optimal allocation of MEs in ATN. Simple extension of problem formulation [6] to ATN gives the following formulation:

Given ATN with N-M nonleaf nodes. Allocate $D=N-M$ MEs in the nodes of the ATN (allowing only one ME to be located in each node) in a way providing the maximal system reliability.

In many cases, even for $D=N-M$, greater reliability can be achieved if some of MEs are gathered in the same position providing redundancy (in hot standby mode) and some positions remain empty, than if all the MEs are evenly distributed between all the nonleaf nodes.

In order to take into account the possibility of reliability improvement by uneven ME distribution and to consider a general case in which number of MEs is not necessary equal to the number of nonleaf nodes, we extend the optimal ME allocation problem as follows:

Find allocation of arbitrary number D of MEs with given state probability distributions (depending on MEs allocation) which maximizes reliability of ATN with given topology.

3. Model Description

An ATN can be represented by acyclic directed graph $G=(C,E)$ with N nodes $c_i \in C$ ($1 \le i \le N$), M of which are leaf ones. The nodes are numbered in such a way that for any arc $(ci,cj) \in Ej>i$ and last M numbers are assigned to the leaf nodes: $c_{N-M+1},\dots,c_N$ (note that such numbering is always possible in acyclic directed graph). The existence of arc $(ci,cj) \in E$ means that a signal can be transmitted directly from node i to node j. One can define for each nonleaf node c_i a set of nodes Λ_i directly following c_i: $cj \in \Lambda_i$ *if* $(ci,cj) \in E$.

There are D available MEs with different characteristics. Each ME has its unique number. Multistate elements located in each nonleaf node c_i ($1 \le i \le N-M$) can transmit a signal to the nodes belonging to the set Λ_i. In each state k, these elements transmit a signal to some subset λ_{ik} of Λ_i (in the case of total failure, the ME cannot transmit a signal to any node: $\lambda_{ik}=\varnothing$). Each ME d ($1 \le d \le D$) located at c_i can have K_i different states and each state k has probability $p^d{}_{i\lambda_{ik}}$, such that $p^d{}_{i\lambda_{ik}}$. The states of all the MEs are independent. One can see that for each ME d all its possible state probability distributions depending on its location are defined by matrix $P^d=\{p^d{}_{i\lambda_{ik}}\}$, $1 \le i \le N-M$, $1 \le k \le K_i$.

Note that one can define the same set of possible states for each ME located at certain node c_i. Indeed, if some ME n can provide connection from c_i to a set of nodes λ_{ik} and ME m cannot provide this connection, the state corresponding to set λ_{ik} can be defined for both MEs, while $p^n{}_{i\lambda_{ik}}\neq 0$ and $p^m{}_{i\lambda_{ik}}=0$.

A signal can be retransmitted by the ME located at c_i only if it reaches this node.

The system reliability R is defined as a probability that a signal generated at the root node c_1 reaches all the M leaf nodes $c_{N-M+1},\dots,c_N$.

The ME allocation problem can be considered as a problem of partitioning a set Δ of D MEs into a collection of N-M mutually disjoint subsets Δ_i ($1\le i\le N$-M), i.e. such that

$$\bigcup_{i=1}^{N-M}\Delta_i=\Delta, \qquad (1)$$

$$\Delta_i \cap \Delta_j=\emptyset,\ i\neq j, \qquad (2)$$

Each set Δ_i, corresponding to ATN position c_i, can contain from 0 to D MEs. The partition of the set Δ can be represented by the vector $H=\{h(d), 1\le d\le D\}$, where $h(d)$ is the number of the subset to which ME d belongs. One can easily obtain the cardinality of each subset Δ_i as

$$|\Delta_i|=\sum_{d=1}^{D}1(h(d)=i). \qquad (3)$$

For the given vector H one can obtain state probability distribution of each ME d allocated at node $c_{h(d)}$ from matrix P^d as $P^d{}_{h(d)\lambda_{k(d)}k}$ for $1\le k\le K_{h(d)}$.

For the given ATN topology (C,E) and for the given state probability distributions of the MEs P^d ($1\le d\le D$), the only factor influencing the ATN reliability is the allocation of its elements H. Therefore, the optimal allocation problem can be formulated as follows.

Find vector H^* maximizing the ATN reliability R:

$$H^*(C,E,P^1,\dots,P^D)=$$

$$=\arg\{R(H,C,E,P^1,\dots,P^D)\to\max\} \qquad (4)$$

4. ATN Reliability Estimation Based on a Universal Generating Function

The procedure used in this paper for network reliability evaluation is based on the universal generating function (also called u-function) technique, which was introduced in [7] and which proved to be very effective for reliability evaluation of different types of multi-state systems [8-14]. The u-function extends the widely known ordinary moment generating function.

4.1. U-function for Individual MEs

The UGF (u-function) of a discrete random variable X is defined as a polynomial

$$u(z)=\sum_{k=1}^{K}q_k z^{X_k}, \qquad (5)$$

where the variable X has K possible values and q_k is the probability that X is equal to X_k.

In order to represent random sets of ATN nodes that receive a signal, we modify the UGF by replacing the random value X with the random binary vector $V=\{v(1)\dots v(N)\}$ such that $v(j)$ corresponds to node c_j.

Consider a multistate element d located at position c_i. In each state k ($1\le k<K_i$), the ME provides a signal transmission from c_i to a set of nodes λ_{ik}. In order to represent the set λ_{ik}, we determine vector V_{ik} as follows

$$v_{ik}(j)=\begin{cases}1, & c_j\in\lambda_{ik}\\ 0, & c_j\notin\lambda_{ik}\end{cases}. \qquad (6)$$

The polynomial

$$u_{id}(z)=\sum_{k=1}^{K_i}p^d{}_{i\lambda_{ik}}z^{V_{ik}} \qquad (7)$$

represents all the possible states of the ME located at c_i by relating the probabilities of each state k to the value of a random vector V_i (representing set λ_{ik}) in this state.

Note that the absence of any ME at position c_i implies that no connections exist between c_i and any other position. This means that any signal reaching c_i cannot be retransmitted in this node. In this case, the corresponding u-function takes the form

$$u_{i0}(z)=z^{V_0} \qquad (8)$$

where $v_0(j)=0$ *for* $1\le j\le N$.

4.2. U-Function for Group of MEs Allocated at the Same Position

Consider two MEs n and m allocated at the same position c_i. Assume that first ME is in state s and second one is in state g. The probability of this composition of states is $p^n{}_{i\lambda_{is}}\cdot p^m{}_{i\lambda_{ig}}$. A signal generated by the two MEs reaches all the nodes belonging to set $\lambda_{is}\cup\lambda_{ig}$. This set can be represented by vector $V_{is}\oplus V_{ig}$, where the $\oplus$ operator (logical OR) for two arbitrary vectors A and B is defined as follows:

$$a(j)\oplus b(j)=\begin{cases}0 & if\ \ a(j)=b(j)=0,\\ 1 & otherwise\end{cases} \quad (1\le j\le N). \qquad (9)$$

In order to obtain the u-function of a subsystem consisting of two MEs n and m located at the same position c_i, a composition operator Ω is introduced. This operator determines the u-function for a group of MEs using simple algebraic operations on the individual u-functions of the MEs. The composition operator for a pair of MEs n and m takes the form:

$$\Omega(u_{in}(z),u_{im}(z))= =\Omega(\sum_{k=1}^{K}p^{n}{}_{i\lambda_{ik}}z^{V_{ik}},\sum_{f=1}^{K}p^{m}{}_{i\lambda_{if}}z^{V_{if}})= =\sum_{k=1}^{K_i}\sum_{f=1}^{K_i}p^{n}{}_{i\lambda_{ik}}p^{m}{}_{i\lambda_{if}}z^{V_{ik}\oplus V_{if}}. \qquad (10)$$

The resulting polynomial relates probabilities of each of the possible combinations of states of the two MEs (obtained by multiplying the probabilities of corresponding states of each ME) with vectors representing sets of nodes reseiving the signal in the given combination of states.

One can see that the operator Ω satisfies the following conditions:

$$\Omega\{u_1(z), \ ,u_t(z),u_{t+1}(z), \ ,u_v(z)\}= =\Omega\{\Omega\{u_1(z), \ ,u_t(z)\},\Omega\{u_{t+1}(z), \ ,u_v(z)\}\} \qquad (11)$$

for arbitrary t. Therefore, it can be applied in sequence to obtain the u-function for an arbitrary group of MEs allocated at c_i:

$$\tilde{U}_i(z)=\underset{d\in\Lambda_i}{\Omega}(u_{id}(z))=\sum_{k=1}^{\tilde{K}_i}\tilde{q}_{ik}z^{\tilde{V}_{ik}}, \qquad (12)$$

where $\tilde{K}_i$ is a number of different states of the group of MEs allocated at c_i, $\tilde{q}_{ik}$ is the probability that only the nodes belonging to the set represented by the vector $\tilde{V}_{ik}$ receive the signal directly from c_i. One can consider the group of MEs allocated at c_i as a single ME with state distribution (12).

4.3. U-Function for the Entire ATN

Assume that a signal generated by MEs located at c_1 in state s reaches c_2 ($c_2\in\lambda_{1s}$ which corresponds to $\tilde{v}_{1s}(2)=1$). If the MEs located at c_2 are in state g, the signal generated at c_2 reaches all the nodes belonging to λ_{2g}. Therefore, when the first group of MEs is in state s and the second group is in state g, the set of nodes receiving the signal is $\lambda_{1s}\cup\lambda_{2g}$. This set can be represented by vector $\tilde{\mathbf{V}}_{1s}\oplus\tilde{\mathbf{V}}_{2g}$.

If a signal generated at c_1 at some state s does not reach c_2 ($c_2\notin\lambda_{1s}$ which corresponds to $\tilde{v}_{1s}(2)=0$), the group of MEs located at c_2 cannot retransmit the signal in any of its states and, therefore, MEs located at c_2 don't affect the state of the ATN. The set of nodes receiving the signal remains λ_{1s} represented by the vector $\tilde{\mathbf{V}}_{1s}$. In the general case of arbitrary states of the two groups of MEs, one can use the following function ω to determine the random vector $\hat{\mathbf{V}}_2$ representing the set of nodes receiving the signal from c_1 directly or through c_2:

$$\hat{\mathbf{V}}_2=\omega(\tilde{\mathbf{V}}_1,\tilde{\mathbf{V}}_2)=\begin{cases}\tilde{\mathbf{V}}_1, & \tilde{v}_1(2)=0,\\ \tilde{\mathbf{V}}_1\oplus\tilde{\mathbf{V}}_2, & \tilde{v}_1(2)=1.\end{cases} \qquad (13)$$

To represent all the possible combinations of states of the two groups of MEs, one has to relate the corresponding probabilities (obtained by multiplying the probabilities of corresponding states of each ME group) with the values of the random vector $\hat{\mathbf{V}}_2$ in these states. For this purpose, we introduce a composition operator Ψ over u-functions of groups of MEs located at c_1 and c_2:

$$\hat{U}_2(z)=\Psi(\tilde{U}_1(z),\tilde{U}_2(z))= \Psi(\sum_{s=1}^{\tilde{K}_1}\tilde{q}_{1s}z^{\tilde{\mathbf{V}}_{1s}},\sum_{g=1}^{\tilde{K}_2}\tilde{q}_{2g}z^{\tilde{\mathbf{V}}_{2g}})= \sum_{s=1}^{\tilde{K}_1}\sum_{g=1}^{\tilde{K}_2}\tilde{q}_{1s}\tilde{q}_{2g}z^{\omega(\tilde{\mathbf{V}}_{1s},\tilde{\mathbf{V}}_{2g})}=\sum_{k=1}^{\hat{K}_2}\hat{q}_{2k}z^{\hat{\mathbf{V}}_{2k}} \qquad (14)$$

The resulting polynomial $\hat{U}_2(z)$ represents the probabilistic distribution of the possible values of the random vector $\hat{\mathbf{V}}_2$ corresponding to set of nodes receiving the signal from c_1

directly or through the MEs located at c_2. The random vector $\hat{\mathbf{V}}_2$ can have $\hat{K}_2 \le \tilde{K}_1\tilde{K}_2$ different values. The probability of each state k of group of MEs located at c_1 and c_2 is $\tilde{q}_{2k}$.

Consider a random vector $\hat{\mathbf{V}}_i$ representing a set of nodes receiving the signal directly from c_1 or through the MEs located at $c_2, \dots, c_i$. It can easily be seen that the addition of the MEs located at c_{i-1} changes the set of nodes receiving the signal in such a way that the random vector $\hat{\mathbf{V}}_{i+1}$, representing this new set, takes the form:

$$\hat{\mathbf{V}}_{i+1} = \omega(\hat{\mathbf{V}}_i, \tilde{\mathbf{V}}_{i+1}) = \begin{cases} \hat{\mathbf{V}}_i, & \hat{v}_i(i+1) = 0, \\ \hat{\mathbf{V}}_i \oplus \tilde{\mathbf{V}}_{i+1}, & \hat{v}_i(i+1) = 1. \end{cases} \quad (15)$$

Let $\hat{U}_i(z)$ be the u-function representing probabilistic distribution of $\hat{\mathbf{V}}_i$. Since node c_{i+1} cannot receive the signal from any node c_m with $m>i+1$, the probability that the signal generated at c_1 reaches c_{i+1} is completely determined by $\hat{U}_i(z)$. Therefore, we can obtain a recursive expression for the u-function representing the distribution of ATN states:

$$\hat{U}_{i+1}(z) = \Psi(\hat{U}_i(z), \tilde{U}_{i+1}(z)) = \Psi\left(\sum_{k=1}^{\hat{K}_i} \hat{q}_{ik} z^{\hat{\mathbf{V}}_{ik}}, \sum_{f=1}^{\tilde{K}_{i+1}} \tilde{q}_{i+1\ f} z^{\tilde{\mathbf{V}}_{i+1\ f}}\right) = \sum_{k=1}^{\hat{K}_i}\sum_{f=1}^{\tilde{K}_{i+1}} \hat{q}_{ik}\tilde{q}_{i+1\ f}\, z^{\omega(\hat{\mathbf{V}}_{ik}, \tilde{\mathbf{V}}_{i+1\ f})} = \sum_{k=1}^{\hat{K}_{i+1}} \hat{q}_{i+1\ k}\, z^{\hat{\mathbf{V}}_{i+1\ k}} \quad (16)$$

where $\hat{K}_{i+1} \le \hat{K}_i\tilde{K}_{i+1}$.

Note that for any $\hat{U}_i(z)$ and $\tilde{U}_{i+1}(z) = u_{i+1\ 0}(z)$ (corresponding to empty position c_{i+1})

$$\hat{U}_{i+1}(z) = \Psi(\hat{U}_i(z), u_{i+1\ 0}(z)) = \hat{U}_i(z). \quad (17)$$

One can obtain the u-function representing the distribution of the ATN states when all the MEs are considered (or, equivalently, the probabilistic distribution of random vector $\hat{\mathbf{V}}_{N-M}$) applying the Eq. (16) in sequence for $i=1$, $i=2,\dots$, $i=N-M-1$. Summing probabilities $\hat{q}_{N-M\ k}$ for all the states k in which $v_{N-M\ k}(j) = 1$ for $N-M+1 \le j \le N$, one obtains the probability that the signal reaches all the leaf nodes, which is equal to ATN reliability index.

4.4. Simplification of U-Functions

Observe that when u-function $\hat{U}_i(z)$ is obtained, the values $\hat{v}_{ik}(1), \dots, \hat{v}_{ik}(i)$ representing the presence of a signal at nodes $c_1, \dots, c_i$ are not used further for determining $\hat{U}_m(z)$ for any $m>i$. Indeed, when determining $\hat{U}_{i+1}(z)$, we need to know only the probabilities that the signal reaches nodes $c_{i+1}, \dots, c_N$. It does not matter through what paths the signal reaches these nodes. For example, if the signal reaches c_{i+1} through a number of different paths (represented by the same number of different terms in $\hat{U}_i(z)$), one does not have to distinguish these paths. The only thing one has to know is the sum of probabilities of states in which these paths exist, meaning that one can collect the corresponding terms in $\hat{U}_i(z)$ by replacing all the values $\hat{v}_{ik}(1), \dots, \hat{v}_{ik}(i)$ in vectors $\hat{\mathbf{V}}_{ik}$ of $\hat{U}_i(z)$ with zeros and collecting the like terms.

If in some state k $\hat{v}_{ik}(i+1) = \dots = \hat{v}_{ik}(N) = 0$, the signal cannot reach any position from c_{i+1} to c_N independently of states of MEs located in these positions. Therefore, this state does not contribute to signal propagation to the leaf nodes and the corresponding term can be removed from the u-function $\hat{U}_i(z)$.

Taking into account the above-mentioned considerations, one can drastically simplify polynomials $\hat{U}_i(z)$ for $1 \le i \le N-M$ using the following operator $\varphi(\hat{U}_i(z))$ which

- zeroes $\hat{v}_{ik}(1), \dots, \hat{v}_{ik}(i)$ in each term of $\hat{U}_i(z)$ ($1 \le k \le \hat{K}_i$);
- removes all the terms in which $\hat{\mathbf{V}}_{ik}$ contain only zeros;
- collects like terms in the resulting polynomial.

4.5. Algorithm for Determination of ATN Reliability

Using the UGF technique described above, one can obtain the ATN reliability for the given set of parameters ($p^d_{i\lambda_{ik}}$, λ_{ik}) $1\le i\le N\text{-}M$, $1\le k\le K_i$, $1\le d\le D$ and the given ME allocation H applying the following procedure, which is convenient for numeric implementation:

1. Determine vectors V_{ik} corresponding to sets λ_{ik} for the positions $c_1,\ldots,c_{N\text{-}M}$ using rule (6).
2. For each ME d located at position $h(d)$ determine the u-function $u_{h(d)d}(z)$ using expression (7) with probabilities $p^d_{h(d)\lambda_{h(d)k}}$ for each state k.
3. Obtain u-functions $\tilde{U}_i(z)$ for each nonempty node c_i using expression (12) and Ω operator (10). For empty nodes j assign $\tilde{U}_j(z) = u_{j0}(z)$, where $u_{j0}(z)$ is defined in (8).
4. Assign $\hat{U}_1(z) = \tilde{U}_1(z)$.
5. Apply expression

   $$\hat{U}_{i+1}(z) = \Psi(\varphi(\hat{U}_i(z)),\tilde{U}_{i+1}(z)) \text{ for}$$

   i=1,2,..., N-M-1 in sequence using operator Ψ (14) and operator φ described in the previous section.
6. Simplify polynomial $\hat{U}_{N\text{-}M}(z)$ using operator φ and obtain the ATN reliability R as the coefficient of the term of $\varphi(\hat{U}_{N\text{-}M}(z))$ in which $\hat{v}_{N\text{-}M}(j)=1$ for all $N\text{-}M+1\le j\le N$.

Note that in the general case, the resulting polynomial contains 2^M-1 terms. Therefore, the suggested method can be applied for ATNs with moderate values of M.

5. Optimization Technique

Finding the optimal ME allocation in ATN is a complicated combinatorial optimization problem having $(N\text{-}M)^D$ possible solutions. An exhaustive examination of all these solutions is not realistic even for a moderate number of positions and MEs, considering reasonable time limitations. As in most combinatorial optimization problems, the quality of a given solution is the only information available during the search for the optimal solution. Therefore, a heuristic search algorithm is needed which uses only estimates of solution quality and which does not require derivative information to determine the next direction of the search.

The recently developed family of genetic algorithms is based on the simple principle of evolutionary search in solution space. GAs have been proven to be effective optimization tools for a large number of applications. Successful applications of GAs in reliability engineering are reported in [9-22].

It is recognized that GAs have the theoretical property of global convergence [23]. Despite the fact that their convergence reliability and convergence velocity are contradictory, for most practical, moderately sized combinatorial problems, the proper choice of GA parameters allows solutions close enough to the optimal one to be obtained in a short time.

5.1. Genetic Algorithm

Basic notions of GAs are originally inspired by biological genetics. GAs operate with "chromosomal" representation of solutions, where crossover, mutation and selection procedures are applied. "Chromosomal" representation requires the solution to be coded as a finite length string. Unlike various constructive optimization algorithms that use sophisticated methods to obtain a good singular solution, the GA deals with a set of solutions (population) and tends to manipulate each solution in the simplest manner.

A brief introduction to genetic algorithms is presented in [24]. More detailed information on GAs can be found in Goldberg's comprehensive book [25], and recent developments in GA theory and practice can be found in books [22, 23]. The steady state version of the GA used in this paper was developed by Whitley [26]. As reported in [27] this version, named GENITOR, outperforms the basic "generational" GA. The structure of steady state GA is as follows:

1. Generate an initial population of N_S randomly constructed solutions (strings) and evaluate their fitness. (Unlike the "generational" GA, the steady state GA performs the evolution search within the same population improving its average fitness by replacing worst solutions with better ones).
2. Select two solutions randomly and produce a new solution (offspring) using a crossover procedure that provides inheritance of some basic properties of the parent strings in the offspring. The probability of selecting the solution as a parent is proportional to the rank of this solution. (All the solutions in the population are ranked by increasing order of

their fitness). Unlike the fitness-based parent selection scheme, the rank-based scheme reduces GA dependence on the fitness function structure, which is especially important when constrained optimization problems are considered [28].Allow the offspring to mutate. Mutation results in slight changes in the offspring structure and maintains diversity of solutions. This procedure avoids premature convergence to a local optimum and facilitates jumps in the solution space. The positive changes in the solution code created by the mutation can be later propagated throughout the population via crossovers.

3. Decode the offspring to obtain the objective function (fitness) values. These values are a measure of quality, which is used in comparing different solutions.
4. Apply a selection procedure that compares the new offspring with the worst solution in the population and selects the one that is better. The better solution joins the population and the worse one is discarded. If the population contains equivalent solutions following the selection process, redundancies are eliminated and, as a result, the population size decreases. Note that each time the new solution has sufficient fitness to enter the population, it alters the pool of prospective parent solutions and increases the average fitness of the current population. The average fitness increases monotonically (or, in the worst case, does not vary) during each genetic cycle (steps 2-5).
5. Generate new randomly constructed solutions to replenish the population after repeating steps 2-5 N_{rep} times (or until the population contains a single solution or solutions with equal quality). Run the new genetic cycle (return to step 2). In the beginning of a new genetic cycle, the average fitness can decrease drastically due to inclusion of poor random solutions into the population. These new solutions are necessary to bring into the population new "genetic material" which widens the search space and, like a mutation operator, prevents premature convergence to the local optimum.
6. Terminate the GA after N_c genetic cycles.

The final population contains the best solution achieved. It also contains different near-optimal solutions, which may be of interest in the decision-making process.

5.2. Solution Representation and Basic GA Procedures

To apply the genetic algorithm to a specific problem, one must define a solution representation and decoding procedure, as well as specific crossover and mutation procedures.

As it was shown in section 2, any arbitrary D-length vector H with elements $h(d)$ belonging to the range $[1,N\text{-}M]$ represents a feasible allocation of MEs. Such vectors can represent each one of the possible $(N\text{-}M)^D$ different solutions. The fitness of each solution is equal to the reliability of ATN with allocation, represented by the corresponding vector H. To estimate the ATN reliability for the arbitrary vector H, one should apply the procedure presented in section 3.

The random solution generation procedure provides solution feasibility by generating vectors of random integer numbers within the range $[1,N\text{-}M]$. It can be seen that the following crossover and mutation procedures also preserve solution feasibility.

The crossover operator for given parent vectors $P1$, $P2$ and the offspring vector O is defined as follows: first $P1$ is copied to O, then all numbers of elements belonging to the fragment between a and b positions of the vector $P2$ (where a and b are random values, $1 \le a < b \le D$) are copied to the corresponding positions of O. The following example illustrates the crossover procedure for $D=6$, $N\text{-}M=4$:

$$P1=2\ 4\ 1\ 4\ 2\ 3$$
$$P2=1\ 1\ 2\ 3\ 4\ 2$$
$$O=2\ 4\ 2\ 3\ 4\ 3$$

The mutation operator moves the randomly chosen ME to the adjacent position (if such a position exists) by modifying a randomly chosen element $h(d)$ of H using rule $h(d)=max\{h(d)\text{-}1,1\}$ or rule $h(d)=min\{h(d)+1,N\text{-}M\}$ with equal probability. The vector O in our example can take the following form after applying the mutation operator:

$$O=2\ 3\ 2\ 3\ 4\ 3.$$

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