MULTIRESPONSE REGRESSION MODELS FOR DIGITAL IMAGE PROCESSING

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ANNOTATION

Multiresponse regression model basics for digital image processing are described. The models are used for color correction and restoration. Computational procedures for model discrimination, coefficient estimation, and model adequacy verification are presented taking into consideration digitization of the color coordinates.

1. INTRODUCTION

Digital image can be presented as a set of color pixels and color of a pixel is described by color coordinates vector $Y=\{y_j\}$, where $j=1,K$, m - number of color coordinates defined by color model used for the image (for instance RGB or Lab). Color correction process represents the transformation of the initial image pixels color coordinates accordingly to correction function. This function is known as tone curve in image processing programs and is used for color and tone correction of images [1]. However this method of color correction does not take into consideration the probabilistic nature of color observation errors.

Relation between the true pixel color coordinates X and the color coordinates of the distorted image Y can be represented by means of multiresponse regression model [2] in the form of

$$Y=F(B,X)+E, \qquad (1)$$

where $X=\{x_1,x_2,\cdots,x_m\}^T$ - the color coordinates of a pixel for undistorted image; $Y=\{y_1,y_2,\cdots,y_m\}^T$ - the observed pixel color coordinates for distorted image; $E=\{e_1,e_2,\cdots,e_m\}^T$ - normally distributed error of observation with the expectation vector $E\{E\}=0$ and covariance matrix $E\{EE^T\}=V_E$; $F(B,x)=\{f_1(B,x),\cdots,f_m(B,x)\}^T$ - vector of predefined functions accurate within coefficients; $B=\{b_1,b_2,\cdots,b_k\}^T$ - vector of unknown coefficients.

When estimates $\hat{B}$ of the coefficients are known, the color coordinates for undistorted image can be calculated as $X=F^{-1}(\hat{B},Y)$, where $X=F^{-1}(B,Y)$ is the inverse function to $Y=F(B,X)$.

2. COEFFICIENT ESTIMATION

An experiment is used for a model construction when the color samples with known color coordinates are observed together with the image. The results of the experiment are used for defining the most credible model type and coefficient estimation. Then the model is used for pixel color coordinates converting for the entire image.

Maximum likelihood estimates $\hat{B}$ of the coefficients B provide maximum to the likelihood function logarithm in the form of

$$L(\hat{B})=\max_B\left\{-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{a_i}\left[Z_{ij}{}^T W_i{}^T V_E{}^{-1} W_i Z_{ij}\right]\right\} \qquad (2)$$

where $Z_{ij}=Y_{ij}-F(B,X_i)$, Y_{ij} – color of a_i pixels observed on the sample, X_i – a priori known color of the sample, n- number of the color samples used, W_i – color's weights, which define the importance of the observation error for different colors.

The solution of the extremum problem (2) can be achieved by iteration algorithm

$$\hat{B}^s=\hat{B}^{s-1}+\rho^{s-1}V_B{}^{-1}\sum_{i=1}^{n}a_i\Phi(X_i)\widetilde{V}^{-1}\overline{Z}_i\left(\hat{B}^{s-1}\right) \qquad (3)$$

where n – number of color samples, ρ - iteration step,

$$\widetilde{V}^{-1}=W^T V_E{}^{-1}W\ ;\ \overline{Z}_i=\overline{Y}_i-F(B,X_i),$$

$$\Phi(X)=\left\{\frac{\partial f_1(B,X)}{\partial B}\mid\hat{B},\cdots,\frac{\partial f_m(B,X)}{\partial B}\mid\hat{B}\right\}$$

For each iteration s iteration step value ρ should provide increasing of the objective function $L(B)$ to avoid the iteration process divergence. The covariance matrix of the coefficient estimates is

$$V_B==\left[\sum_{i=1}^{n}a_i\Phi^T(X_i)\widetilde{V}_i^{-1}\Phi(X_i)\right]^{-1}$$

The elements of vector $\overline{Y_i}$ are

$$\bar{y}_1 = \sum_{i=1}^{k_1} y_{1i} \sum_{j=1}^{k_2} \sum_{l=1}^{k_3} p_{ijl}, \quad \bar{y}_2 = \sum_{j=1}^{k_2} y_{1i} \sum_{i=1}^{k_1} \sum_{l=1}^{k_3} p_{ijl},$$

$$\bar{y}_3 = \sum_{l=1}^{k_3} y_{1i} \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} p_{ijl},$$

where k_j (j=1,2,3) (for 3 coordinate color model) - number of grouping intervals for a color coordinate, p_{ijl}- probability of color coordinates Y hitting grouping interval (i,j,l), i,j,l=1,K,256 for 24 bit color.

3. Models Discrimination

The models discrimination method is based on the likelihood ratio logarithm in the form of

$$\Delta^{rg} = \sum_{j=1}^{n} L^r{}_j - \sum_{j=1}^{n} L^g{}_j,$$

where $L^r{}_j$, j=12,K,n- likelihood function logarithm for model r and for color sample j. The likelihood function logarithm for a sample is

$$L_i' = \sum_{j=1}^{a_i} L_j = -\frac{1}{2} \sum_{j=1}^{a_i} \left[Z_{ij}{}^T W_i{}^T V_E{}^{-1} W_i Z_{ij} \right].$$

A mean likelihood function logarithm

$$\bar{L} = \frac{1}{n} \sum_{i=1}^{n} L_i$$

serves as a discrimination criterion.

The more $\bar{L}$ is the more credible the corresponding model will be. Taking into account matrix multiplication formula an expression for likelihood function logarithm for one pixel can be presented as:

$$2L = -\sum_{l,t=1}^{m} d_{lt} y_l y_t + \sum_{l,t=1}^{m} d_{lt} y_l (f_t + f_l) - \sum_{l,t=1}^{m} d_{lt} f_l f_t, \qquad (4)$$

where $D = \tilde{V}^{-1} = \{d_{lt}\}$.

Likelihood function logarithm for one sample is calculated as:

$$2L_j' = -\sum_{l,t=1}^{m} d_{lt} \sum_{i=1}^{a_j} y_l y_t + \sum_{l,t=1}^{m} d_{lt} (f_t + f_l) \sum_{i=1}^{a_j} y_l - a_j \sum_{l,t=1}^{m} d_{lt} f_l f_t.$$

The last expression allows to calculate the likelihood function logarithm for different models if the following sums are known:

$$\lambda_l' = \sum_{i=1}^{a_j} y_{il} \text{ and } \lambda_{lt}'' = \sum_{i=1}^{a_j} y_{il} y_{it}.$$

Taking into consideration color coordinates grouping we will get

$$\lambda_1' = \sum_{i=1}^{k_1} y_{1i} \sum_{j=1}^{k_2} \sum_{l=1}^{k_3} p_{ijl},$$

$$\lambda_{11}' = \sum_{i=1}^{k_1} y_{1i}{}^2 \sum_{j=1}^{k_2} \sum_{l=1}^{k_3} p_{ijl},$$

$$\lambda_{12}' = \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} y_{1i} y_{2j} \sum_{l=1}^{k_3} p_{ijl}.$$

Then the likelihood function logarithm for one sample can be presented as:

$$2L_j' = -\sum_{l,t=1}^{m} d_{lt} \lambda_{lt}'' + \sum_{l,t=1}^{m} d_{lt} \lambda_l' [f_t(B,X) + f_l(B,X)] - a_j \sum_{l,t=1}^{m} d_{lt} f_t(B,X) f_l(B,X). \qquad (5)$$

Using λ_1, λ_2, λ_3 values which do not depend on a model type and are calculated for a color sample only once likelihood function logarithms for different models can be obtained.

The mean value of the likelihood function logarithm for one sample is

$$\bar{L}_j = \frac{1}{a_j} L_j'. \qquad (6).$$

The mean value of the likelihood function logarithm for n color samples on the image is

$$\bar{\bar{L}} = \frac{1}{\sum_{i=1}^{n} a_j} \sum_{j=1}^{n} L_j' = \frac{1}{\sum_{j=1}^{n} a_j} \sum_{j=1}^{n} a_j \bar{L}_j,$$

where L' value is likelihood function logarithm for one color sample (6).

Likelihood function logarithm variance for one color sample can be presented in the following way:

$$s_{Lj}{}^2 = \frac{1}{a_j - 1} \sum_{i=1}^{a_j} \left[L_i - \bar{L}_j \right]^2 = \frac{1}{a_j - 1} \left[\sum_{i=1}^{a_j} L_i{}^2 - a_j (\bar{L}_j)^2 \right], \qquad (7)$$

where L_i is defined as (4) and $\bar{L}_j$ as (6).

Let us denote the following sums as:

$$\eta'_{111} = \sum_{i=1}^{k_1} y_{1i}^{3} \sum_{j=1}^{k_2} \sum_{l=1}^{k_3} p_{ijl} ,$$

$$\eta'_{112} = \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} y_{1i}^{2} y_{2j} \sum_{l=1}^{k_3} p_{ijl} ,$$

$$\eta''_{1112} = \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} y_{2i} y_{1i}^{3} \sum_{l=1}^{k_3} p_{ijl} .$$

Then we present the sum from equation (7) to calculate the likelihood function logarithm variance for one color sample the following way:

$$\begin{aligned} 4\sum_{i=1}^{a} L_i^2 &= \frac{1}{2} \sum_{l,t,v,w}^{m} d_{lt} d_{vw} \eta''_{ltvw} - \\ &-2 \sum_{l,t,v,w}^{m} d_{lt} d_{vw} (f_w + f_v) \eta'_{ltv} + \\ &+2 \sum_{l,t,v,w}^{m} d_{lt} d_{lt} f_l f_t \lambda''_{lt} - \\ &-2 \sum_{l,t,v,w}^{m} d_{lt} d_{vw} f_l f_t (f_w + f_v) \lambda'_l + \\ &+ \sum_{l=1}^{m} d_{lt} d_{vw} (f_l + f_t)(f_w + f_v) \lambda''_{lt} + \\ &+ \left(a_j \sum_{l,t}^{m} d_{lt} f_l f_t \right)^2 . \end{aligned}$$

Likelihood function logarithm variance $\breve{s}_L^{\,2}$ for the entire image can be presented as:

$$\breve{s}_L^{\,2} = \frac{\sum_{j=1}^{n} (a_j - 1) s_{Lj}^{\,2}}{\sum_{j=1}^{n} a_j - 1} + \frac{\sum_{j=1}^{n} L_j^{\,2} - \left(\bar{\bar{L}}\right)^2}{\sum_{j=1}^{n} a_j - 1} \qquad (8)$$

with number of degrees of freedom

$$f = \sum_{j=1}^{n} a_j - 1 .$$

The first item in the expression (8) presents average weighted likelihood function logarithm variance inside the color samples and the second item is the variance between the color samples.

Proof of hypothesis that model r is more credible than model g consists in calculating of t criterion as:

$$t = \frac{\bar{\bar{L}}^r - \bar{\bar{L}}^g}{\sqrt{\left(\breve{s}_L^{\,2}\right)^r + \left(\breve{s}_L^{\,2}\right)^g}} \sqrt{f} .$$

If $|t| > t_{q,f}$, where $t_{q,f}$ – Student's distribution fractile for significance level q with f number of degrees of freedom, the hypothesis is being accepted.

In case of $|t| \le t_{q,f}$ the difference between the models is nonsignificant and preference is given to less complicated model.

4. Adequacy Verification

Having obtained coefficient estimates it is necessary to verify model data fit. A model may be considered adequate if the residual errors can be explained as observation errors. Maximum likelihood estimates are based on the assumption that the observation errors E have normal distribution, zero expectation value and covariance matrix of V_E. When the coefficient estimates are known another estimate $\hat{V}_E$ of the covariance matrix may be calculated. Adequacy verification consists in proof of the hypothesis of this two matrices equality.

If the observation errors have covariance matrix V_E, then the residual errors have covariance matrix of $\left(1 - \frac{l}{mn}\right) V_E$, where l- number of coefficient estimates, m- number of the model responses. In this câse the mean residuals vector

$$\bar{E} = \frac{1}{n} \sum_{i=1}^{n} E_i$$

should have covariance matrix of $\frac{1}{n}\left(1 - \frac{l}{mn}\right) V_E$.

Then statistics

$$\lambda = \frac{mn^2}{mn - l} \bar{E} V_E^{-1} \bar{E}$$

obeys Pearson's distribution χ_m^2.

In case of unknown covariance matrix V_E its estimate in the form of

$$S = \frac{1}{n-1} \sum_{i=1}^{n} (E_i - \bar{E})(E_i - \bar{E})^T$$

can be used instead. If the hypothesis that mean residuals are equal zero is valid then statistics

$$\lambda = \frac{n(n-m)}{m(n-1)} \bar{E}^T S^{-1} \bar{E}$$

obeys Fisher's distribution $F_{m,n-m}$.

5. INVERSE PREDICTION

The color correction procedure is represented by inverse prediction problem when it is necessary to estimate the vector of pixel color coordinates X for undistorted image by means of the observed pixel color coordinates Y for distorted image and defined function

$$Y = F(\hat{B}, X),$$

where $\hat{B}$ - estimates of the coefficients, obtained on the stage of model construction.

That is estimates $\hat{X}$ of color coordinates are defined by equation

$$\hat{X} = F^{-1}(\hat{B}, Y), \tag{9}$$

where Y- observed color coordinates of the image

The solution of the last equation for nonlinear problem (9) can be achieved by iteration algorithm in the form of

$$\hat{X}^s = \hat{X}^{s-1} + \rho^{s-1}\left[\left(\Omega^{-1}\right)^{s-1}\Delta^{s-1}\right], \tag{10}$$

where ρ- iteration step, and $\Delta^s = Y - F(\hat{B}, \hat{X}^s)$.

Covariance matrix V_X of estimates $\hat{X}$ (10) is

$$V_X = [\Omega^{-1} V_Y (\Omega^{-1})^T],$$

where

$$\Omega = \left\{ \frac{\partial f_1(\hat{B}, X)}{\partial X} \mid \hat{X}, \dots, \frac{\partial f_m(\hat{B}, X)}{\partial X} \mid \hat{X} \right\}.$$

6. CONSIDERATIONS ABOUT MODEL TYPES AND REFERENCE COLORS

Depending of the nature of the digital images the color correction process can be divided into three categories: picture reproductions obtained inside the premises and usually using artificial light; digital photographs (or scanned) taken outside the premises and usually under natural light and scanned faded photographs.

6.1. Picture Reproductions

In this case only color correction is usually necessary. Useful model type is a cubic polynomial for each color coordinate that is 9 to 15 coefficients in model (1). Color samples with known color coordinates are used for model coefficients estimation.

6.2. Digital Photographs

For the images obtained by means of digital camera (or scanned photographs) the main goal is to provide a realistic image which has psychologically correct or desirable color hue for certain image elements.

Usually the tone correction is necessary first for this kind of images. The objectives are: to set white and black image points and to set the appropriate lightness for all tone zones – midtones, halftones, shadows and lights.

After tone correction goes color correction, but instead of color samples in this case cognate colors are used like color of human skin which should have color coordinates over the range 75L(9 - 12)a(23 -25)b in Lab color model, green grass and leaves: 50L(- 10 - 20)a(20-35)b, blue sky: (75 - 60)L(0 -10)a(- 8 - 40)b and the like.

Polynomial models can be used as well as spline polynomial for independent correction of tone zones.

6.3. Faded Photographs

There are different phenomena effecting the color shifts with time and because of that there are different possible nonlinear regression-time models describing how color coordinates change with time. The model type is determined in advance during preliminarily research of the fading process of different photo materials. These models help roughly restore color pattern of the initial image.

The next correction steps are the same as in paragraph 5.2. Color coordinates prior fading are determined subjectively by visual analysis of the image. This analysis is based on use of psychological colors and on a priopi color information of some image elements.

7. CONCLUSION

Using the multiresponse regression models allows to carry out color correction and restoration on the basis of statistical approach to observed image colors which provides much more substantial ground for such kind of image transformation.

The proposed correction method provides substantially higher color restoration accuracy and flexibility than in case of tone curve method.

REFERENCES

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