FREQUENCY ASSIGNMENT PROBLEN AUTOMATION

I. Charamisinau

South Methodist University, 3160 SMU Blvd, Dallas, TX, 75275, USA, charamisinau@yahoo.com

ABSTRACT

This paper dwells upon frequency assignment problem in radio communication systems. Some ways of solving the problem of network frequency assignment are considered that could reduce the computational efforts when solving this problem in different statements.

Different approaches for solving frequency assignment problem are discussed: the methods based on random search, graphs coloring, genetic algorithms. The results of program realization and testing proposed algorithms of frequency assignment are cited.

1. INTRODUCTION

For the last few years we are facing vast expansion of cellular communications. Last year the number of the cell phone users was about 300 million and according to the estimations by the year 2004 this number will increase to 1 billion and overshoot the number of users of conventional phones [1].

Such a growth of cell phones requires better radio frequency assignment and usage that could be not an easy task because up-to-date cellular systems counts some times hundreds of base stations.

The frequency assignment problem consists in distributing frequency channels among transmitter stations in such a way to avoid any electromagnetic interference. The problem is reduced to assigning some frequencies to each of stations ensuring prescribed frequency spacing between channels in two different stations.

This paper dwells upon frequency assignment automation algorithms.

2. THE PROBLEM OF FREQUENCY ASSIGNMENT

Following are input data for different frequency assignment tasks.

1. The permissible frequency arrays (or channels) that give frequencies allowed using for each transceiver station.

The matrix of permissible frequencies is a Boolean matrix with as many rows as possible frequencies exist and columns as stations. An entry (*ij*) of the matrix is 1 if *i*-th frequency could be assigned to *j*-th station and the entry (*ij*) equals to 0 otherwise.

2. The channel numbers vector. Its length equals to the total number *m* of the stations for which we are doing frequency assignment. The *i*-th entry of the channel numbers vector equals to the number of channels required for *i*-th station.
3. The constraint matrix. It shows minimal possible frequency spacing between any channels for a pair of different stations.

The constraint matrix is a square matrix that has as many rows and columns as stations exist. Its (*ij*)-th entry shows minimal possible frequency spacing between channels in *i*-th and *j*-th stations. It is clear the matrix is symmetric with respect to the main diagonal.

The frequency spacing between channels in two different stations depends on the distance between them, power of their transmitters, susceptibility of their receivers, relief, buildings and so on. If some of these parameters change it is necessary to correct frequency spacing and therefore the constraint matrix.

The constraint matrix could be determined precisely and approximately. It could be determined precisely using software that uses geographic maps and the relief to determine coupling between two stations and approximate calculations could be done using flat surface model (like EPM-73 or Akamura-Hata model).

The optimal solution for network frequency assignment problem would be a set of frequencies (channels) for each station, that

- satisfies the constraint matrix;
- includes minimal number of different frequencies.

The network frequency assignment could be done in three ways.

1. Specify the station network as a nonperiodical structure. This requires specifying allowed frequencies and the constraint matrix for the whole network.
2. Specify the station network as a periodical structure, that is formed from clusters of the

same type. They are usually groups of 3, 4, 5, 7 or 9 cells [3]. Than the network frequency assignment of the entire network is reduced to the frequency assignment of a single cluster. In this case the constraint matrix is specified for the only cluster taking into account periodical repetition of the cluster. The constraint matrix in this case becomes small but densely filled and often it does not contain zeros at all.

The main advantage of this approach is that instead of network frequency assignment for the great number of stations we need set frequencies just for the only relatively small cluster or even use plans designed for simple clusters by companies that produce equipment for cellular communication systems.

The drawback of this approach is that it is not always possible to place stations in order to get regular structure.

3. It is impossible to specify the station network as a periodical structure. But there exists a periodical network structure that is close enough to given irregular network. So we can do the frequency assignment for the periodical network using the second approach. Then additional calculations should be performed for the places with the greatest irregularity where given irregular network differs from periodical one.

It is done using the results obtained for periodical structure as a seed value. In this case the constraint matrix is specified only for stations from the irregular part of the network. Some frequencies should be excluded from the permissible frequency arrays, that should be done to match the frequencies that are already set for stations from surrounding regular part.

This approach allows avoiding the drawback of the second approach.

3. Computational Complexcity Estimation

It could be shown that even the simplest case of the network frequency assignment when the constraint matrix is a Boolean one requires graph coloring that is NP-hard problem [5]. Thus the exact solution of the frequency assignment problem (providing minimal number of different frequencies to be used) can be obtained only by exhaustive search, that requires a lot of time. Let us derive estimation for the exhaustive search time.

The number of possible frequency values in all stations of the network is:

$$N = \prod_{i=1}^{m} C_{ni}^{ki} \qquad (1)$$

here n_i and k_i are the numbers of frequencies allowed and required respectively for i-th station.

The formulae (1) shows that the number of possible variants of the frequency assignment is really huge that is why it is more efficient to use approximate algorithms. Let us discuss three possible ways of the problem solution.

4. Random Search

This approach is based on random assigning frequencies to stations. The goal is to find the frequency distribution matrix. This matrix is similar to the matrix of permissible frequencies, but the number of ones in each row is equal to the number of frequencies that should be assigned to corresponding station.

The frequency distribution matrix could be found from the matrix of permissible frequencies by excluding the proper numbers of ones from its rows. Ones to be excluded are chosen randomly.

The obtained matrix can be or cannot be a solution. We can verify it by checking whether the differences between the frequency values satisfy the constraint matrix.

If the frequency distribution matrix satisfies the constraint matrix then we have one of the possible solutions of the network frequency assignment problem, if not then we should generate a new frequency distribution matrix.

Algorithms based on this approach are simple, but they do not take into account previously obtained variants that were already tested. Therefore the algorithm converges to solution very slowly and is practically useless for the frequency assignment of complex networks.

5. Graph Theoretic Formulation Of The Problem

Let us represent a network of stations by means of a graph $G=(V,E)$ whose nodes $v_i \in V$ are in one-to-one correspondence with the stations of the network considered and the set E of edges is defined as $E=\{e=(v_i,v_j)/c^j_i=1\}$, where c^j_i are entries of the Boolean constraint matrix C: $c^j_i \in C$. I.e. only a pair of the nodes is connected by an edge corresponding to a pair of stations that should have some frequency spacing.

When the problem is to get frequency spacing satisfying the constraint matrix we introduce the graph $G=(V,E,M)$ with marked edges. The mark $m_k \in M$ of the edge $e_k=(v_{ki},v_{kj}) \in E$ equals to (i,j)-th entry of the constraint matrix. And the frequency assignment problem can be reduced to the graph nodes coloring where the colors are considered as frequencies. Further when we need to assign several frequencies to a single station we should assign the same number of colors to the proper node of the graph.

The node coloring of the graph is called permissible if it satisfies the following conditions:

1. every node $v_i \in V$ has the same number of colors as the appropriate station should have channels;
2. for every $v_i \in V$ the set $P(v_i)$ of its colors is the subset of the permissible frequency array allowed to use for appropriate station;
3. for every $e_k=(v_{ki},v_{kj}) \in E$ the condition $|p(v_{ki})-p(v_{kj})| \geq m_k$ takes place for any pair of colors $p(v_{ki}) \in P(v_{ki})$ and $p(v_{kj}) \in P(v_{kj})$.

Being in practical use graph coloring method consists in sequential assigning of colors to the nodes and placing limitations on the colors of nodes that are connected with the considered ones. Some heuristics [2] could be proposed to determine the sequence of colorings. One of them is based on sequencing nodes by their degrees.

The first node will be the one of maximal degree (it corresponds to the line of the constraint matrix with the maximal number of nonzero elements). A randomly selected subset of colors allowed by the corresponding constraint matrix line (the power of the subset is equal to the number of channels required) is assigned to this node. Then in the same way the next uncolored node is chosen that is a neighbor to colored ones.

This algorithm is more complex than the above-considered one. But it converges to solution faster then random search algorithm. If a solution exists it will be found though it may require examining a lot of possibilities.

6. Frequency Assignment Using Evolutionary Programming

Evolutionary programming is one of the types of genetic algorithms [4].

Initial population is formed by random generation of a set of the frequency distribution matrixes. The number of generated matrixes is the initial population size (maximal population size). Then we chose some matrixes (minimal population size) from initial population that provides the minimum of the error function – they satisfy better to the constraint matrix.

Mutation of the matrixes chosen can be done by changing some elements of the frequency distribution matrixes. The simplest types of the mutations are

1. changing a row of the frequency distribution matrix that corresponds to correction of the frequencies chosen for one station;
2. changing a column of the frequency distribution matrix that corresponds to correction of one of the frequencies for all stations.

All matrixes of the minimal population are mutated in several times so that total number of the matrixes generated by means of their transformations becomes equal to initial population size.

After mutations we check whether one of the mutated matrixes meets the constraint matrix. If none of new matrixes represents a solution than we choose again a minimal population and repeat the mutations until any solution is found.

Though this algorithm does not give 100% guarantee of finding the solution, but it is as a rule more effective than any enumeration algorithm. It converges to solution quicker than any algorithm of random search and has less probability of solution loss.

7. Experimental Verification of the Proposed Algorithms

The proposed algorithm of frequency assignment based on the evolutionary programming was programmed and tested. When testing it has been compared with the simple random search algorithm of the solution of the frequency assignment problem.

Numerical experiments shown that evolutionary programming algorithm is the most effective for minimal population size of 10 and initial (maximal) population size of 70-140. It turned out that the initial population size has a little effect on algorithm efficiency if it is not too close to minimal population size.

Reduction of the minimal population size greatly improves the algorithm effectiveness (its running speed). But if it is less than 10 the algorithm becomes unstable: sometimes it finds the solution quickly but often its speed is reduced tens times or it even fails to converge to a solution.

The explanation could be that for a small minimal population size the algorithm could do more iterations in a less time, but the whole population could fall into a local minimum and the probability of finding a way-out is too little having a small minimal population size.

Having optimal population size settings the evolutionary programming algorithm checked up to 300000 at the average. A random search algorithm does not converge to a solution even after 10000000 iterations.

Let us consider the task of the frequency assignment for a real network. It has regular structure formed by three three-sector cells shown in fig. 1.

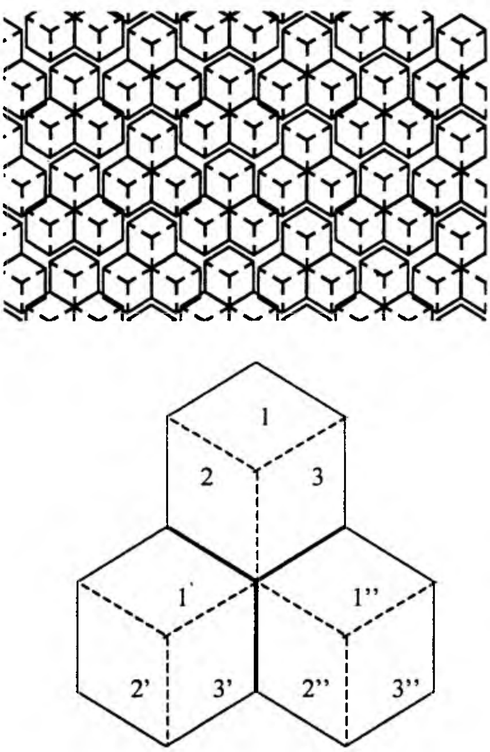

Figure 1. Regular structure formed by three-sector cells

Each station has three antennas, each antenna covers 120° sector of hexagonal cell. The frequency plan is calculated for cluster formed by tree tree-sector stations. The cluster repeated in space to cover all territory (see fig. 1).

The constraint matrix of this network can be given as the following matrix:

The *i*-th line of the matrix resents the set of frequency spacing values between transmitters of the *i*-th antenna (120° sector). In our case antennas from the same cell should have frequency spacing 3, any pair of antennas from neighboring sectors should have frequency spacing 2, any pair of antennas from the sectors separated by one sector should have frequency spacing 1. In our case there will be no sectors separated by two or more sectors so there will be no zeros in the matrix.

$$C = \begin{bmatrix} 3 & 3 & 3 & 1 & 2 & 1 & 1 & 1 & 2 \\ 3 & 3 & 3 & 1 & 1 & 2 & 2 & 1 & 1 \\ 3 & 3 & 3 & 2 & 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 3 & 3 & 3 & 1 & 2 & 1 \\ 2 & 1 & 1 & 3 & 3 & 3 & 1 & 1 & 2 \\ 2 & 1 & 1 & 3 & 3 & 3 & 2 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 & 2 & 3 & 3 & 3 \\ 1 & 1 & 2 & 2 & 1 & 1 & 3 & 3 & 3 \\ 2 & 1 & 1 & 1 & 2 & 1 & 3 & 3 & 3 \end{bmatrix}$$

For each sector it was specified average number of 36 allowed channels (out of 112 channels) and it was necessary to choose 4 frequencies for each sector. Complete enumeration will require $8.5*10^{42}$ steps according to the formulae (1).

References

[1]. Maneshin V.V. Key ways of the development of the mobile cellular communications. *Mobile systems.* **2**, 2000 (in Russian).

[2]. Pisarenko G.K. Network frequency assignment algorithms in telecommunication systems Information technologies. №6, 2000. pp. 18-25 (in Russian).

[3]. Aladin I.M., Dejurni I.I. FDMA: New options. Mobile systems. №1, 2000. pp. 24-30

[4]. Kurejchik V.M. Genetic algorithms. State. Problems. Perspectives. *Proc. of academy of science. Control systems and theory.* **1**, 1999. pp.144-160 (in Russian).

[5]. Garey M.R. and Johnson D.S. Computers and Intractability: a Guide to the theory of NP-completeness. Freeman, San Francisko, Ca., 1979.