Minsk, Belarus, October 2-4, 2001

FUZZY-BASED MULTICRITERIAL OPTIMIZATION OF ECONOMIC AND ECOLOGICAL SYSTEM

L. Dymova, T. Chegerova[1], D. Sevastianov and P. Sevastianov[2]

[1] - Belorussian Research Institute of Ecopathology and Occupational Diseases, 70, Vitebsky Ave. 212004, Mogilev, BELARUS. tatyana@mail.telecom.mogilev.by.
[2] - Mogilyov State Engineering University, ave. Mira, 43, 212005, Mogilyov, BELARUS.

ABSTRACT

The technique is offered and the specialized software for the solving the problem of multicriteria optimization of an ecological state (*ES*) and state of health of the population (*SHP*) taking into account the restrictions of a financial and economic and social character is developed. The description of the software realizing a multicriteria evaluation of *ES* and *SHP* and optimization of expenditures on their improving is given. As an example the variant of problems solution for money optimal distribution of resources given from in the Mogilyov city budget on improving the state of *ES* and, consequently, *SHP* is considered. As a result of the software work, the optimum solution for distribution of resources for clearing measures and struggle with each contamination component in atmosphere is obtained separately at defined level of the budget.

The state of population health (*SHP*) is reflected by complex of phenomena in an environment. Therefore evaluation of an ecological state (*ES*) of regions for acceptance of adequate administrative solutions on improving *SHP* is an important problem, the solution of which needs application of modern mathematical techniques.

Traditionally evaluation of *ES* of inspected regions implies the database creation, which based on measured factors of radiation and chemical contamination of environment, radiation - ecological map-making of a district.

The evaluation of a state of health of the population is carried out either on common morbidity rate (or morbidity rate on classes of diseases), or at mass of screening medical-preventive inspections on average for inspected population values of measured criteria describing the state of separately taken system of an organism (for example, haemogramm parameters). At the same time, the state of health is an integrated concept which is characterized by the multiple factors. For development of adequate medical-preventive and administrative solutions in region *ES* and *SHP* must evaluated on a set of private criteria, such, for example, as concentration of heavy metals, ions, organic substances etc., the part from which can be measured with some degree of accuracy (concentration etc.), and the part is given in the verbal form (for example, radiofobia at the population or its sanitary-hygienic culture). Each criteria obviously or implicitly generates an appropriate private criterion *ES* or *SHP*, the formalization of which is not trivial at all.

The marked difficulties can be overcome by introduction of generalized criterions *ES* and *SHP* as convolutions of private criteria taking into account the factors of their relative importance. To this purpose it is necessary to decide problems of adequate mathematical formalization of private criteria, defined at a qualitative level, problems of ranging of large number of criteria at an initial qualitative (verbal) evaluation of their relative significance and their convolution into a generalized criterion.

In the present paper the structure of private criteria of *ES* and *SHP* was determined on the data submitted to Belorussian Research Institute of Ecopathology and Occupational Diseases by the Mogilyov city center of epidemiology and hygiene. The data on dynamics and multiplicity of excess permissible concentration (*PC*) by basic contaminants of an atmospheric air and on morbidity rate of the population in 1988 - 1998 were included in their structure on data of a information system - "Health" on sample of 65 thousand people.

The following components were included in a structure of basic contaminants of an atmospheric air: carbon desulfide, hydrogen sulfide, methanol, paraxylene, dimethylterphthalates, phenol, sulfur oxides, nitrogen dioxide, nitric oxide, aerosols, carbon oxides, lead, zinc, copper, benzo(a)pyrene.

The datas on morbidity rate of the population includes the following diseases: neoplasmes, rheumatism, hypertension, *CHD*, acute myocardial

infarction, chronic pharyngitis, chronic tonsillitis, pneumonia, chronic bronchitis, bronchial asthma, stomach ulcer, chronic gastritis, gall-stone disease, nephritis, dermatitis.

The statistical processing for 10 years has revealed the statistically significant correlation of morbidity rate of chronic gastritis, chronic bronchitis and dermatitis with a level of a content in an atmospheric air carbon disulfide, hydrogen sulfide and sulfur oxides. In this connection the state of *ES* was evaluated on a content of the following components in an atmospheric air: carbon disulfide, hydrogen sulfide, sulfur oxides.

For a convolution of private criteria into a generalized criterion it was necessary to bring them to a mutual norm. For this purpose the mathematical means of the fuzzy sets theory was used. Let's consider for example such important quantitative parameter, as concentration of hydrogen sulfide (H_2S) in an air *C* (mg/m3). There is *PC* of hydrogen sulfide equal 0.008. The values of $C < PC$ are permissible, and the degree of this permissibility (preference) increase to the decrease of *C*. To formalize such descriptions in fuzzy sets theory the means preference functions is used which vary from 0 in the field of inadmissible values up to a maximum value equal 1, in the field of the best values of quality parameter.

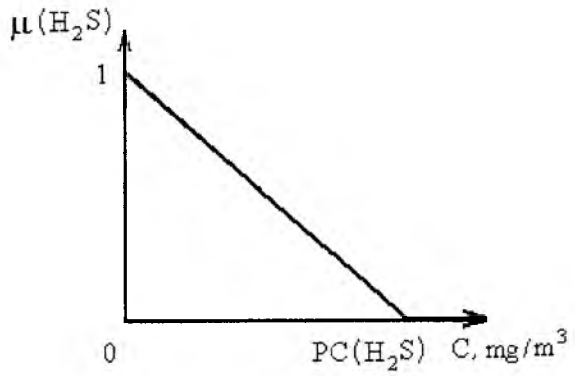

Figure 1. Example preference function for C of hydrogen sulfide in the air

After the preference functions partial criteria describing *ES* and *SHP* are constructed it is necessary to rank them, i.e. to receive coefficients of their relative importance or significances - αi.

For an evaluation of coefficients of relative importance well-known technique of T. Saaty, based on a construction of a matrix of pair comparisons of taken into account partial criteria was used [1].

Let $\alpha_i>0$, $i=1...$, N - absolute ranks of criteria. Let's consider a matrix of relative ranks $A=\{\alpha_i/\alpha_j\}$. Obviously, that if one matrix A on the right on a vector of unknown ranks $W=(\alpha_1, \alpha_2,...,\alpha_N)$, he will receive $AW=NW$. Thus, if the matrix A is known, the determination of W is reduced to a solution of a system of the linear algebraic equations. However, in practice, the elements of a matrix, representing pairwise values of relative importance of criteria, are set on the base of subjective preferences, i.e. are rather inexact. Therefore T. Saaty offers to receive a solution from the equation of an aspect $AW=\lambda W$, where λ - maximum eigenvalue of a matrix A. It is now proved, that the problem of the definition of a vector W can be reduced to a problem of minimization of a functional

$$S=\sum\sum(A_{ij}*\alpha_j-\alpha_i)^2, \qquad (1)$$

at restriction $\sum\alpha_i=1$.

To define the elements of a matrix of conjugate comparisons the linguistic evaluations of pairwise importance of criteria could be connected to a natural numerical series. In a tabl. 1 the provisional set of such linguistic estimations obtained by generalization of experience of problem solving of a decision making is represented.

Table 1. Linguistic evaluation of relative importance of criterions

Close equivalent	1
Are almost equivalent	3
A little bit more preferable	5
Much more preferable	7
Strictly more preferable	9
Intermediate values of importance	2,4,6,8

Let's consider the process of a construction of a matrix of conjugate comparisons on the following example. Let there are three criterions *X*, *Y* and *Z*, and on evaluations of the expert *X* is almost equivalent *Y*, which corresponds number 3 in the scale of linguistic estimations; *Y* is a little bit more preferable *Z* (number 5); *X* it is strictly more preferable *Z* (number 9). The resulting matrix of conjugate comparisons is submitted in a tabl. 2.

Table 2. Example matrix construction of pairwise comparisons

	X	Y	Z
X	1	3	9
Y	1/3	1	5
Z	1/9	1/5	1

In the tabl. 3 the ranks (degree of negative influence on health) considered contaminants of an atmospheric air (carbon desulfide, hydrogen sulfide, sulfur oxides), obtained with the help of matrixes of pairwise comparisons on the base of above-mentioned technique are represented. As it could be seen from the table, the substances which have the first class of hazard according to a list *PC* of the contaminations № 3086-84 from 20.08.84 by Ministry of Health of BSSR receive the greatest ranks.

Table 3. Ranks of basic contaminants of an atmospheric air

carbon disulfide	1.3
hydrogen sulfide	1.3
sulfur oxides	0.4

The calculation of global criterions of quality ES was carried out on expressions which correspond:

To criterion of maximum pessimism (2)

$$D_1(C_1, C_2, C_3)=min(\mu_1 C_1)^{\alpha 1}, \mu_2(C_2)^{\alpha 2}, \mu_3(C_3)^{\alpha 3}),$$

To additive criterion (3)

$$D_2(C_1, C_2, C_3)=(\alpha_1\mu_1(C_1)+\alpha_2\mu_2(C_2)+\alpha_3\mu_3(C_3))/3$$

To multiplicative criterion (4)

$$D_3(C_1, C_2, C_3)=\mu_1(C_1)^{\alpha 1*}\mu_2(C_2)^{\alpha 2*}\mu_3(C_3)^{\alpha 3}$$

Where m_1, m_2, m_3, a_1, a_2, a_3, C_1, C_2, C_3 are functions of preference, ranks and concentration of carbon disulfide, hydrogen sulfide and sulfur oxides accordingly. Global criterions of quality *SHP* were calculated similarly.

In the total on the base of measurements data and expert estimations integrated quantitative evaluations of a condition *ES* and *SHP* were varying due to the technique of constracting similarly to the partial preference functions from 0 up to 1.

On the base of principles, explained in the literature, and microeconomic models, especially on the Rassel-Spofforf's model [2, 3, 4], we developed a model of optimum distribution of the budgetary funds intended for improving of an ecological state of an air environment at a regional level. Thus the specificity available data and modern economic situation, defining basic restrictions were taken into account at accepted for optimum administrative resolutions on protection the population health and environment.

The model is based on existence of obvious direct dependency between expenditures intended on refinement of an atmospheric air, and concentration of polluting substances in it, and also relation between concentration of substances in an air and volumes of their ejections. On fig. 2 the scheme of the developed model is submitted.

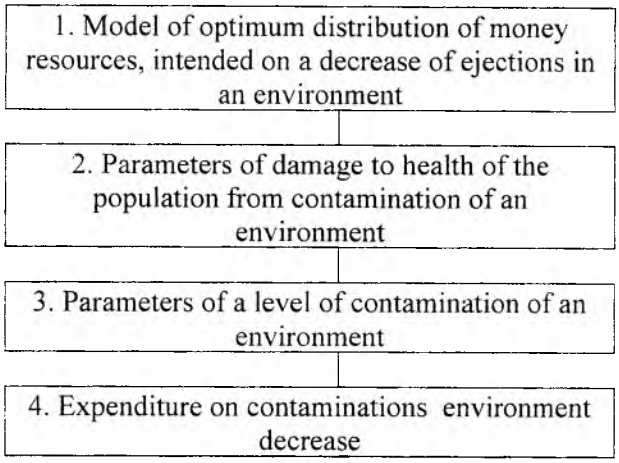

Figure 2. The scheme - diagram of an integrated regional model of environment protection.

For obtaining necessary regression equation the data of Mogilyov city inspection of natural resources about the amount of ejections of carbon disulfide, hydrogen sulfide and sulfur oxides by the enterprises «Chimvolokno», artificial filament plant and heat producers of the Mogilyov thermal networks, and also information about expenditures on their decrease in the period from 1987 to 1999 (in values of 1987).

On the base of these information the so-called wasteful coefficients K_i - that is amount of money resources necessary for a decrease of ejections of each contaminant on one ton were deduced. In the total it is obtained, that for a decrease of ejections of carbon disulfide on one ton it is necessary to expend K_1=1.845 rbl., for a decrease of ejections of hydrogen sulfide on one ton it is necessary to expend K_2=6.289 rbl., for a decrease of ejections of sulfur oxides on one ton it is necessary to expend K_3=0.154 rbl.

On the base of comparisons of available data about ejections with data of a city center of hygiene and epidemiology about concentration of hydrogen sulfide, carbon disulfide and sulfur

oxides in atmosphere, the significant direct statistical connection between ejections - V_i (thousand tons) and concentration of substances in an air - C_i (mg/m^3) is detected:

$$C_1=0.00373+0.00327*V_1, \quad (5)$$

$$C_2=0.00048+0.00186*V_2, \quad (6)$$

$$C_3=0.0007+0.00033*V_3, \quad (7)$$

Where V_1, V_2, V_3 - ejections of carbon disulfide, hydrogen sulfide, sulfur oxides accordingly.

Let's designate expenditures on refinement of an air from separate polluting substances through z_1, z_2..., z_n, where n - amount of observable polluting substances. Let Z – city budget intended on struggle with contaminations of air. Then in our case the restriction will be obvious:

$$\sum_{i=1}^{n} z_i = Z,\ n=3. \quad (8)$$

On data of Mogilyov city inspection of natural resources the ejections of carbon disulfide, hydrogen sulfide, sulfur oxides in volumes 5.154 thousand t, 0.881 thousand t and 18.203 thousand t accordingly were registered in 1999. On the base of these data and designed wasteful factors K_i it is possible to deduce expressions for calculation of final volumes of ejections after realization of ecological measures:

$$V_1=5.154-z_1/K_1/1000;$$
$$V_2=0.881-z_2/K_2/1000;$$
$$V_3=18.203-z_3/K_3/1000,$$

That in turn allows to receive an obvious dependency of concentration of substances in atmosphere from expenditures on their decrease:

$$C_1=0.00373+0.00327*(5.154-z_1/K_1/1000) \quad (9)$$
$$C_2=0.00048+0.00186*(0.881-z_2/K_2/1000) \quad (10)$$
$$C_3=0.0007+0.00033*(18.203-z_3/K_3/1000) \quad (11)$$

Following stage was the definition of an dependency between global criterions of a state of health of the population (D_{SHP}) and atmospheric air (D_{ES}). Thus in calculations the additive forms of global criterions were applied. In an outcome of a statistical analysis we have received following regressive dependency (Criterion Fisher F=29.17, p<0.0006):

$$D_{SHP}= -0.709+2.396*D_{ES}. \quad (12)$$

As D_{SHP} is function of concentration C_1, C_2, C_3, the expression (12) in its turn determines an dependency of a criterion D_{SHP} with these concentration. Substituting in (12) instead of C_1, C_2, C_3, of expression (9) - (11) the generalized criterion of quality of health as function of expenditures of financial assets on improvements of atmosphere in region D_{SHP} (z_1, z_2, z_3) is obtained.

The basic purpose is the decrease of morbidity of the population of city by illnesses connected to contamination of an atmospheric air, by optimum distribution of the city budget intended on struggle with these contaminations. Therefore appropriate problem of optimization at restriction (8) was formulated as follows:

$$(z_1, z_2, z_3)_{opt} = \arg \max_{z_1,z_2,z_3} D_{shp} \quad (13)$$

The solution of optimization problem was found by a method of a random search. Thus using the generator of normally distributed random numbers in an interval from 0 up to Z the random values z_1, z_2, z_3 were selected, then the fulfillment of restriction (8) was evaluated. If the absolute value of a discrepancy $\sum_{i=1}^{n} z_i - Z$ did not exceed 10 % from Z, from expressions (9) - (11) the concentration of substances in an air were determined and from expression (3) the additive criterion of quality of an atmospheric air D_{ES} was calculated. Further from (12) an additive global criterion of a condition of health of the population D_{SHP} was determined.

After realization according to the scheme a large series of calculations, we have received an array of values of criterions D_{SHP}, from it we have selected a maximum value D_{SHP}. Values z_1, z_2, z_3 which define it were a solution of optimization problem.

This technique is realized as the software in Builder C ++.

References

[1] Saaty T. Scaling Method for Priorities in Hierarhical Structures, J.of *Mathematical Psychology*. -1977.-V.15, **3**.- P.234-281.

[2] Niz A. Economyka I okruzhayuschaya sreda. - M.: *Ekonomica*. - 1981.

[3] Voronovicky N.M., Gofman K.G., Gusev A.A., Spivak V.A. Ekonomicheskie osnovi plati za zagryaznenie okruzhayuschey prirodnoy sredi. – *Economica* I matematicheskie metodi, 1975, т. XI, v. 3.

[4] Lave L., Seskin R. Air Pollution and Human Health, *Science*, August 1970