CREATION OF DECISION SUPPORT SYSTEM ON THE BASES OF NON-LINEAR MODEL OF THE ECONOMY

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ANNOTATION

The article is dedicated to the creation of decision support system, which is based on non-linear dynamic model of the economy of the Republic of Belarus. It is the model that uses a variety of assumptions of synergetic economics. It extends the traditional economics, the assumptions of which can be out of effectiveness in transition period.

1. INTRODUCTION

In transition economy the decision-making process on the economy-wide level becomes particularly topical in macroeconomic adjustment, analysis, and planing. It requires a large volume of data using a variety of computer software, combining methods of mathematical economics and computer science. It is high time to say that the most of existent informational and analytical systems use balanced or optimized models with a linear nexus that makes medium- and long-run forecasting impossible due to their linearity. However, the practice affirms that transition processes could be more adequately described using non-linear dynamic models, which allow obtaining a synergetic effect. The creation of the decision support system on the basis of non-linear model is a representative of one of the first in the newer generation of informational and analytical systems.

There are many good foreign economic and mathematical models of the country economy, which could be of great interest for us. As far as we are concerned we should create the model that will embody the specific features of our economy reflecting the main priorities of the country, its national, ethnic, demographic, and governmental interests. The elaborated non-linear dynamic model takes into account the real and financial sector of economy as well as the international sector. As for primary information we have chosen the Murphy Model [2] that is created for a small country dealing with more highly-developed one. The choice of the Murphy Model is not occasional because it uses a variety of assumptions, which are very similar to the economy of the Republic of Belarus, namely:

- Belarus is a small country;
- the foreign-currency prices of imports;
- the real rate of return in the world market.

2. THE ENTERPRISE PRODUCTION BLOCK OF THE MODEL

We commence with the enterprise production block. Broadly speaking, it has two roles. The first is to specify a competitive equilibrium of inputs and outputs to which the economy converges in the long run. The second is to provide medium-run neoclassical equilibrium which act like moving target that influences the dynamics of many of variables of the model.

The first-order conditions may be approached either via the production and transformation functions, or via their duals – the cost and revenue functions. Given the assumption of constant returns to scale, the composition of the cost-minimizing input mix at any output level depends only on relative input prices, and not on the level of output: similarly, the optimal proportions for the output bundle depend only on ratios of output prices, and not on the scale of production. Hence the relevant duals are the unit cost and revenue functions.

The structure of the enterprise production block is summarized pictorially in fig. 1.

On the first level domestic factors are modeled using Cobb and Douglas production function that has two inputs (labour and capital) and determines production possibilities:

$$F^{MR}=A_1 K_B^{\alpha} E_B^{\beta}$$

where α and β – the coefficients of capital and labour elasticity respectively.

Except domestic goods, imported commodities are also sold at the domestic market. The second level shows the distribution of consumption of gross output between domestic and imported goods. That distribution can be expressed by CES function. There are tight economic relations between Belarus and Russia, thus the innovation is the import disaggregation into import from Russia and import from the West Europe. Summarizing, we obtain the following equations:

The elasticity of substitution between import from Russia and import from West Europe and aggregate domestic factor in the production of gross output is:

$$\sigma_i = \frac{1}{1+\rho_i}, i = \overline{1,2} \qquad (5)$$

The cost functions dual to (2) and (3) are respectively:

$$p_B^{MR1} = \left(A_2^{\sigma_1} p_{MRus}^{\rho_1\sigma_1} + A_3^{\sigma_1} (p_F^{MR})^{\rho_1\sigma_1}\right)^{1/(\rho_1\sigma_1)}, \qquad (6)$$

$$p_B^{MR2} = \left(A_4^{\sigma_2} p_{MW}^{\rho_2\sigma_2} + A_5^{\sigma_2} (p_F^{MR})^{\rho_2\sigma_2}\right)^{1/(\rho_2\sigma_2)} \qquad (7)$$

Fig. 2 illustrates diagrammatically the links between variables for the determination of output of domestic goods Y_D. In the panel A the price p_X of exports is found from unit revenue p_B and the price p_Y of the domestic good. In the panel B the price ratio p_X/p_Y and capacity Y_B determine output Y_D of the domestic good.

Now we are going to obtain output of the domestic good equation analytically. The CET production function that allow modeling gross output distribution between domestic goods and exports takes the form:

$$Y_B = \left(A_6 X^{-\rho_\tau} + A_7 Y^{-\rho_\tau}\right)^{-1/\rho_\tau}, \qquad (8)$$

where the elasticity of transformation τ is given by:

$$\tau = \frac{1}{1+\rho_\tau}. \qquad (9)$$

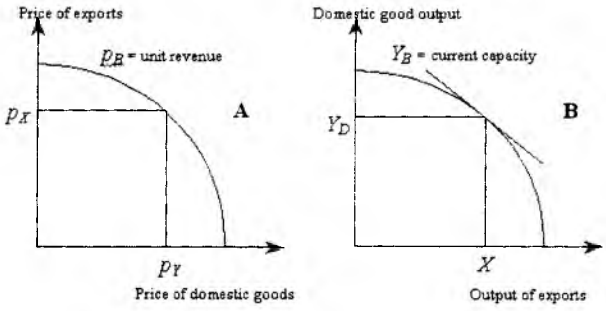

Figure 2. Determination of output of the domestic good

The unit revenue function dual to (8) is:

$$p_B = \left(A_6^{\tau} p_X^{\rho_\tau\tau} + A_7^{\tau} (p_Y^{MR})^{\rho_\tau\tau}\right)^{1/(\rho_\tau\tau)} \qquad (10)$$

Solving for p_X/p_Y, we obtain:

$$\frac{p_X}{p_Y} = \left(\frac{\left[\frac{p_B}{p_Y}\right]^{\rho_\tau\tau} - A_7^{\tau}}{A_6^{\tau}}\right)^{1/(\rho_\tau\tau)} \qquad (11)$$

Then we find the margin rate of transformation (MRT). To obtain the latter, we partially differentiate (8) with respect to X at a fixed value of Y_B and setting the result to the negative price ratio. We get:

$$Y_D = Y_B^{MR}\left(\frac{p_Y^{MR}}{p_B^{MR} A_7}\right)^{-\tau} \qquad (12)$$

$$Y_D = Y_B^{MR}\left(\frac{p_Y^{MR}}{p_B^{MR} A_7}\right)^{-\tau} \qquad (13)$$

Note that export volume is given by $X=Y_D*(X/Y_D)$, and substituting (12) and then (11) using (8), we obtain output of the domestic good equation:

The same way we can use to obtain import and export equations. Write down their final equations:

Fig. 3 shows a graphic illustration of obtaining import from Russia equation.

$$M_{Rus} = F^{MR}\left(\frac{p_{MRus} A_3}{p_F^{MR} A_1}\right)^{-\sigma_1} \qquad (14)$$

$$M_W = F^{MR}\left(\frac{p_{MW} A_4}{p_F^{MR} A_5}\right)^{-\sigma_2} \qquad (15)$$

$$X^{MR} = Y_E^{MR}\left(\frac{p_X}{p_F^{MR} A_6}\right)^{-\sigma} \quad (16)$$

In the panel *A* the price of aggregate factors p_F is found from the values of p_{MRus} and p_B, using the unit cost function. With the ratio $-p_{MRus}/p_F$ given, the value of F is sufficient to determine capacity Y_B and M_{Rus}, as in the panel *B*.

The price indexes are modeled in the long-run the same way as in the medium-run, however, the resulting indicator is the long-run rental price on business capital r_B^{LR} (fig. 4).

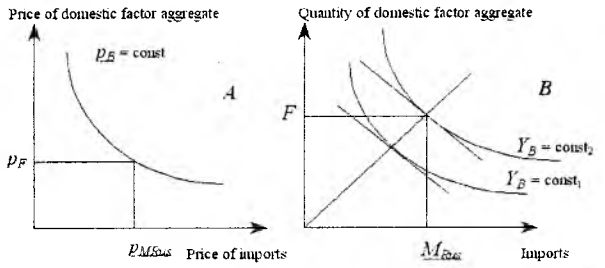

Figure 3. Determination the volume of imports

Let r_B^{ALR} be the gross rate of return, then,

$$r_B^{A_{LR}} = r_B^{LR} / p_Y \quad (17)$$

Hence, the enterprise production block in the long-run can be shown as in fig. 5 (r_B^R is required rate of return).

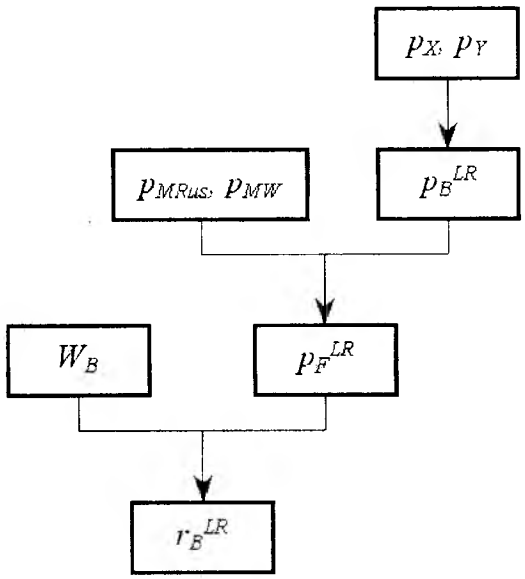

Figure 4. Determination of the rental price of business capital in the long-run

According to Tobin's *q* theory, long-run closure of the enterprise production block converges to the comparison of the long-run gross rate of return with required rate of return. If they are equal, that means that the model has reached its equilibrium. Hence, write down the enterprise production block stochastic dynamic equation, which also describes the business fixed investment:

$$\begin{aligned}\frac{I_B}{K_B} &= \alpha_0 + \alpha_1 \frac{I_B(-1)}{K_B(-1)} + \\ &+ \alpha_2\left(r_B^{A_{LR}}(-2) - r_B^R(-2) - \delta_B\right) + \\ &+ \alpha_3 \ln\left(\frac{p_Y^{MR}(-3)}{p_Y(-3)}\right) + \\ &+ \alpha_4\left(r_S(-4) - r_L(-4)\right) + \\ &+ (1-\alpha_1)(\hat{\gamma} + \delta_B) + \varepsilon,\end{aligned} \quad (18)$$

where:

- I_B – real business fixed investment;
- δ_B – depreciation rate;
- r_S, r_L – short-term and long-term interest rate;
- $\hat{\gamma}$ – medium-run value for natural real growth factor;
- α_0 ... α_4 – estimated coefficients;
- *-1, -2, -3, -4* – lags with respective periods;
- *s*– stochastic error.

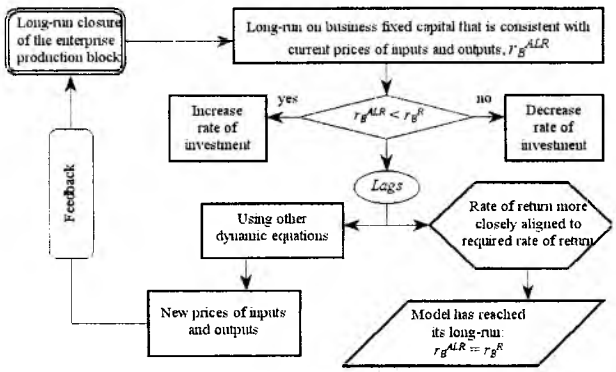

Figure 5. Feedback of the long-run closure of the enterprise production block

3. Financial Sector Of The Model

Get down to financial sector, in which the interest rate, inflation rate, and exchange rate are determined. Important in this determination are:

1. an interest-sensitive money demand function;
2. the operation of uncovered interest parity.

Unlike other parts of the model, the equations of the financial sector are solved under the assumption of rational expectations.

Consider the relations between short-term and long-term interest rates. All of them are expressed as proportion per quarter. Long-term interest rate is 10 years or 40 quarters.

Let $\hat{r}_L(t, \tau)$, $\hat{r}_S(t, \tau)$ be the value expected from the viewpoint of quarter τ to be the expectation prevailing in quarter t $(t \geq \tau)$ for the long-term (short-term) interest rate. The expectation theory of the term structure implies that an investor at $\tau=0$ with expected long-term interest rate $\hat{r}_L(0, 0)$ will be indifferent between holding a long-term security with this expected yield and a sequence of 40 quarterly holdings with expected yields

$\hat{r}_S(0, 0)$, $\hat{r}_S(1, 0)$, $\hat{r}_S(2, 0),\ldots,\hat{r}_S(39, 0)$, provided:

$$1+\hat{r}_L(0,0)=\left[\prod_{t=0}^{39}\left(1+\hat{r}_S(t,0)\right)\right]^{\frac{1}{40}} \qquad (19)$$

Using the approximation that, for $|a| << 1$, the value of $ln(1+a)=a$, hence, (19) may be rewritten as:

$$\hat{r}_L(0,0)=\frac{1}{40}\sum_{t=0}^{39}\hat{r}_S(t,0). \qquad (20)$$

The modified theory of rational expectations uses instead of giving equal weights to expectations of short-term interest rates geometrically declining weights for successfully more and more distant future quarters, followed by zero weights for more distant expectations. In this case, we have the following equation:

$$\hat{r}_L(0,0)=(1-b)\sum_{t=0}^{\infty}b^t\hat{r}_S(t,0), \qquad (21)$$

where $|b|<1$.

To simplify the equation (21), consider the expectation now for expectations of the long-term interest rate one quarter ($t=1$), multiply through by b, and subtract (20) from it. We obtain:

$$\hat{r}_L(0,0)=b\hat{r}_L(1,0)+(1-b)\hat{r}_S(0,0). \qquad (22)$$

It is reasonable to assume that the current short-term interest rate is known, and therefore that $\hat{r}_S(0,0)=r_S$.. Under rational expectations we identify $\hat{r}_L(0,0)$ with the actual market value of the long-term interest rate, namely, r_L. Finally, we identify $\hat{r}_L(1,0)$ with the model's one-period-ahead solution value for the long-term interest rate, namely, $\tilde{r}_L(+1)$. Making these substitutions, (22) becomes:

$$r_L=b\tilde{r}_L(+1)+(1-b)r_S+\theta,$$

where θ is the variable, connected with a sudden changes in expectations about long-term interest rate.

The current domestic inflation rate ξ is measured as the rate of increase of the price of the domestic good:

$$\xi=\Delta\, ln\, p_Y \qquad (24)$$

The relationship between the long-term expected domestic inflation rate and expectations for future short-term inflation rates is modeled analogously to the relationship between long-term and expected short-term interest rates. The coefficient b is determined by value of 0,95. Hence:

$$\tilde{\xi}_{10}=0{,}95\tilde{\xi}_{10}(+1)+0{,}05\xi+\theta. \qquad (25)$$

Determine the notion of *uncovered interest parity*. Interest parity implies the equality between the domestic and foreign interest rate after exchange-rate adjustment, that is:

$$(1+r_S)\frac{\tilde{\psi}(+1)}{\psi}=(1+r_S^F), \qquad (26)$$

where:

- ψ– exchange rate suitable for converting foreign currency,
- r_S^F – nominal short-term foreign interest rate.

Uncovered interest parity means that no risk premia is involved in this calculation.

Money demand is described by the regression equation:

$$r_S=\alpha_0+\alpha_1\, ln(M_2/GNE)+\alpha_2 D+\varepsilon, \qquad (27)$$

where:

- M_2 – stock of currency in circulation,
- GNE – nominal gross national expenditure,
- D – Dummy variable for shift in currency demand.

Obtain the balance of payment equation. Current account deficit on balance of payment includes:

+ interest paid on foreign dollar denominated net Belarusian debt held by foreigner;
+ interest paid on Belarusian ruble denominated net Belarusian debt held by foreigner;

+ net unrequited transfers overseas;
+ dividends paid overseas to foreign equity in Belarusian business capital;
– balance of trade surplus.

There are two ways in which a flow deficit may be settled:

1. by incurring new debt, i.e. by new borrowing overseas (*NBO*);
2. by ceding ownership of domestic real capital to foreigners (so called 'selling the farm').

The latter may be written:

$$p_Y (I^{FOR} - \delta_B K_{FOR}),$$

where:

- I^{FOR} – foreign business fixed real investment;
- K_{FOR} – real stock of foreign-owned business fixed capital.

The balance of payment equation takes the form:

$$\begin{aligned} & NBO + p_Y\left(I^{FOR} - \delta_B K_{FOR}\right) + ABL = \\ & = \frac{\left(D_F^{PRI} + D_F^{PUB}\right)}{\psi} r_F^{AVE} + \left(D_A^{PRI} + D_A^{PUB}\right) r_L^{AVE} + \\ & + \left(R_{12} - R_{14}\right) GDP + \left(\frac{K_{FOR}}{K_B} GOS - \left[\delta_B K_{FOR} p_Y\right]\right) - \\ & - \left(Xp_X - \frac{M_W p^*_{MW}}{EX_W} - \frac{M_R p^*_{MR}}{EX_R}\right), \end{aligned} \tag{29}$$

where:

- ABL – balancing item;
- D_A^{PRI}, D_A^{PUB}, D_F^{PRI}, D_F^{PUB} – private and public sector debt to foreigners contracted in Belarusian rubles and foreign currency;
- r_L^{AVE}, r_F^{AVE} – average interest rate paid on Belarusian debt denominated in Belarusian and foreign currency;
- R_{12} – net unrequited transfers from public sector to overseas as a proportion of nominal GDP;
- R_{14} – ratio to nominal GDP of net unrequited transfers to private sector from overseas;
- GDP – nominal gross domestic product;
- GOS – nominal gross operating surplus of the business sector;
- X – volume of exports;
- p_X – price of exports;
- p^*_{MR}, p^*_{MW} – price of imports in foreign currency (in Russian rubles and other foreign currencies);
- EX_R, EX_W – exchange rate of Russian ruble and other foreign currencies.

4. Conclusion

In conclusion, we have to emphasis that nowadays the questions of the long-run forecasting of the economy of the Republic of Belarus have been raised. During the integration processes the elaborated model lets us choose the optimal trajectories of strategic development, which allow obtaining the maximum economic effect both in real and financial sectors and international economic relations.

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