HIERARCHICAL FUZZY MODEL OF PRODUCTIVE SYSTEM FOR DECISION MAKING SUPPORT PROBLEMS

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ABSTRACT

The management problem for structured system, which is interacting with the environment, considered as the mathematical problem. Fuzzy dynamic model of productive system considered as adequate model. Properties of this model considered. The fuzziness reduction approach for consequences of management decisions analysis proposed.

1. Introduction

One of the fundamental management problems for the modern society development is the management of man-machine systems. Examples of such systems are industrial enterprise, scientific centre, research and development organisation and many others [1].

These real systems have complex internal structure and changing interaction with the environment. Another specific of structured systems is the multidimensional management purposes. I.e. such systems have the set of synchronous purposes and goals as the inherent characteristic.

The existing approaches for management problem solving is based on aggregation of separate mathematical models, with rough and inflexible approximations for quantitative and functional characteristics of the system [1].

However, uncertainty and inconstancy of internal functional relations and fuzziness of environmental influences make solution of management problem very sophisticated. Quantitative information describing the functioning of real systems is fuzzy by the nature. Therefore development of new approaches to the management of such systems is actual [2,3].

With the information technology developments new possibilities to built adequate mathematical models emerged. These models have to take into consideration the inherent uncertainty and multi-dimensionality of real system characteristics. And these modes supposed to be more adequate to complexity and variability of the internal processes in managed systems than the existing models.

In this paper the concept and model of productive system introduced for formal description of wide class of real man-machine systems. Formal approach to the analysis of the model proposed.

2. Productive System With Hierarchic Structure

Real structured systems are functionally related with environment by incoming and outcoming flows of different objects (of substance, energy, information or other nature). In this paper we will consider the class of structured systems, which transform incoming flow of objects into outcoming flow, with certain measurable addition for the characteristics of the system and environment. Such systems we can define as productive [4].

As the application we can consider the economy of certain production enterprise. We assume that formalisation for certain environment can be the set of elementary objects A and relations between them R. For the elements from A and R we can set up conformity with the sets of quantitative characteristics Z and K:

$$Z = \{z_i\}; Z(a) : a \to \Re_+^{M\times N},$$
$$K = \{k_i\}; K(r_j) : r_j \to \Re_+^{M\times J},$$
$$a \subseteq A,\quad i = \overline{1,M} < \infty,\quad j = \overline{1,N} < \infty.$$

Put into consideration discrete time, and define dynamic as the process of changing characteristics of the set A in time.

$$z_t^i(a_j) - z_{t-1}^i(a_j) = \Delta_t z_t^i(a_j) \in \Re; \Delta_t Z_t \in \Re^{M\times N}$$
$$i = \overline{1,M}; j = \overline{1,N}; t > 0 \quad t = 0,1,...T \leq \infty$$

Consider productive system S=<a,r>, $S \subseteq \bar{S}$, $\bar{S} =< A,R >$. Common characteristics of economic systems is the limitedness of material resources. Therefore, determine the principle of conservation of elements from the set A in time:

$$\exists \widetilde{Z} \subseteq Z : \widetilde{Z}_t(A) = C = const(t) \in \Re^{M\times N}, \forall t \quad (1)$$

Characteristics $z \in Z \setminus \widetilde{Z}$, for which the principle of limitedness is not realised, we define as quantitative measures for qualitative characteristics of elements from A.

Interrelation of the system S with the environment we describe by the pair of sub-sets

$$a^0, a^1 \subseteq A : a^j = \{a_t^j\} \subseteq a, 1 \le i \le N$$

$$\Delta_t Z_t(a^j) = -\Delta_t Z_t(A \setminus a), j \in \{0,1\}, t > 0$$

Inside the certain economic system incoming flow of objects (resources) transforms into outcoming flow of objects (products). This transformation we can consider as the functional mechanism of the system. And we can define this mechanism as the set of continual and quasi-linear functions $F(t)$:

$$F_t(a) = \{f_t^j(a)\} : (z(a), k(r)) \to (b_t^i,.., b_t^l)$$

$$j = \overline{1, L} < \infty, 1 \le i, l \le N, t > 0, F_t(a) \in \Re^{L \times N}$$

We consider functioning of system as discrete process, divided by segments

$$[t^0; t^1] : t^0, t^1 > 0,\ 1 < t^1 - t^0 \le \tau \in (1;T)$$

named as cycles of functioning. For each cycle it have to be veritable the balance equation:

$$\sum_{t=t_1}^{t=t_2} \Delta_t Z_t(a^0) = E \times \sum_{t=t_1}^{t=t_2} \Delta_t F(a,t) + \sum_{t=t_1}^{t=t_2} \Delta_t Z_t(a^1),$$

$$E = \{1\} \in \Re^{M \times L}, t \in [t^o; t^1] \subseteq (0;T)$$

System S is productive if changes of all characteristics meet the criteria of added value:

$$\sum_{t=t_1}^{t=t_2} \Delta_t Z_t(a^0) \text{ p } E \times \sum_{t=t_1}^{t=t_2} \Delta_t F + \sum_{t=t_1}^{t=t_2} \Delta_t Z_t(a^1) \quad (2)$$

This criteria means that productive system increases the quantity of qualitative characteristics of the environment (including the system also) $\overline{S} = < A, R >$ on the base of limited resources transformation into products.

Structure of system $S=(a,r)$ is determined by functional units. We can define it as matrix:

$$B_t(S) : (Z_t(a), F(a,t)) \to \Re^{M \times N \times L} \quad (3)$$

$$\Delta_t B_t(S) : (\Delta_t Z_t(a), \Delta_t F(a,t)) \to \Re^{M \times N \times L}$$

Hierarchy of the system's elements we describe as the order relation on the set R:

$$\overline{R} = \bigcup_{j=0}^{H} R_j : R_0 = R(S, \overline{S}) \quad (4)$$

$$R_1 = R(R_2, ..., R_H), \qquad R_H = R(a_1, ..., a_N)$$

Based on certain hierarchy, all set of elements of the system divided to sets of objects. These objects compose different levels of the system.

Functional mechanism of the system $F(t)$ represent as decomposition of the functional $F(S)$ on the different hierarchy levels:

$$F_t^0(S) = \prod_i F_t^{1,i}(Z(a), R_1(a)), i \le N,$$

$$F_t^0(S) = \prod_i \times ... \times \prod_j F_t^{H,j}(Z(a), R_H(a))$$

The environment $\overline{S}$, influences on the characteristics of the system S by the set of factors:

$$\Phi_t = \{\phi_t^j\} : B_t(S) = B_t(S; \Phi_t), j \in J, t > 0$$

Under the objective uncertainty of economic information, all qualitative variables mentioned above we consider as fuzzy numbers [2,3]:

$$z_i = \{\underline{\underline{z}}_i, \underline{z}_i, z_i, \overline{z}_i, \overline{\overline{z}}_i\}; k_i = \{\underline{\underline{k}}_i, \underline{k}_i, k_i, \overline{k}_i, \overline{\overline{k}}_i\}$$

Arithmetical operations with fuzzy numbers based on the generally accepted principle [2]:

$$(\mu * \eta)(x) = \sup_{x=y*z} \{\min\{\mu(y), \eta(z)\}\}$$

$$\mu, \eta : R$$

$$\mu, \eta : R \to [0,1];\ * \in \{\pm, \div, \times\}$$

Thus, on the given definitions and relations we built the fuzzy dynamic model of the productive system S, described as the process of cyclic functioning [3]:

$$B_t(S; \Phi) + \sum_{i=1}^{\tau} \Delta_{t+i} B_{t+i}(S; \Phi) = B_\tau(S; \Phi), \quad (5)$$

$$t > 0,\ 0 < \tau << T$$

On the hierarchy (4) and functional structure (2) we can define the structure of the system as:

$$B_t(S; \Phi) = B_t(Z(a), F_t^0(Z(a), \overline{R}); \Phi),$$

$$B_t(S; \Phi) = \bigcup_i \bigcup_j B_t(Z(a), F_t^i(Z(a), R^j); \Phi)$$

Hence we have fuzzy hierarchical model of productive system.

3. The Multipurpose Control Problem and Hierarchic Productive System

The management problem in real economic systems is multi-purpose oriented. For example, the efficiency versus effectively balancing; and supporting of the stable and robust functioning regime. It can be presented as the multi-criteria management problem [1].

We can define the set of ratios, which reflects functioning of the system S. These ratios defined on quantitative characteristics, and can be interpreted as financial and operational performance ratios:

$$C_t = \{c_t^j\},\ c_t^j : B_t(S) \to (c_1,...,c_p) \subseteq \Re, \quad (6)$$
$$j \in J=\{1,...,P\},\ P<\infty,\ t>0$$

Certainly exists the sub-set of values for these ratios, which reflects the balanced regime of the system S [3]:

$$\hat{C}_t(B_t(S)) = [c_{\min}, c_{\max}] \subseteq \Re^P, t>0 \quad (7)$$

And we can formulate the multi-criteria management problem in terms of dynamic regulation of the ratios under criteria (7).

Economic management is not the full control over the system. Because reactions of system under management actions are not deterministic [1]. One of possible approaches to such management problem is to define the set of management principles. For example we can consider two principles: I) the stable functioning regime sustaining under environmental influences uncertainty, with formal criteria (7); II) supporting the productive regime of the system, with formal criteria (2).

The state of system S in each time described by the structure (3) , which we can represent as:

$$B_t(S) = B_t^{FC}(S) + B_t^{PC}(S) + B_t^{NC}(S)$$

$B_t^{FC}(S), B_t^{PC}(S), B_t^{NC}(S)$ – fully-, partially- and not-controlled characteristics.

Fully-controlled characteristics determined for each functioning cycle, not-controlled characteristics are determined by environmental influences. Partially-controlled characteristics depend on both mentioned above via functional interrelations.

General algorithm of management is:

1. Determination of fully-controlled characteristics for sequenced cycles;
2. Measuring of characteristics and structure of the system after each cycle;
3. Analysis of ratios (6) and reconciliation with criteria (7);
4. Adjusting for fully-controlled characteristics for subsequent cycles.

For solving the certain management problem we need to transform general algorithm into information technology. This technology have to enable the system monitoring, analysis and of characteristics and constructing the management actions in terms of sequences for fully-controlled characteristics.

Proven approach to such information systems is based on three elements for formalisation of structured managed system: Model – Algorithm – Program [3]. In this paper we consider the first part of this triad – model of structured system. On the base of formal model we can formalise the management problem and approach to solving it.

Consider structured productive system as multi-level groups of elements. Hierarchy (4) shows the scaling (or zooming) view on the system. From macro-level: system as an structured object in the environment; to the detail level: elementary objects of the system, which are interrelating and interacting.

For the multi-purpose management problem solution we propose to perform decoupling of principles I) and II) to different levels of hierarchy of the proposed model. This approach resulted with presentation of the general problem as combination of two one-purpose problems, interrelated by characterisation:

$$B_{t+1}(Z, F_{t+1}^0(Z, \bar{R}); \Phi_{t+1}) = U_t^{I,II}(B_t(S_t; \Phi_t)),$$

$$U_t^{I,II}(B_t(S_t;\Phi_t)) = U_t^I \cup U_t^{II},\ t>0 \quad (7)$$

Formal definition for these two one-purpose problems can be following:

For the detail-level of the system:

$$B_{t+1}(Z, F_{t+1}^0(Z, R_H); \Phi_{t+1}) = U_t^{II}(B_t(S_t; \Phi_t))$$

$$\sum_{t=t_1}^{t=t_2} \Delta_t Z_t(a^0) \prec E \times \sum_{t=t_1}^{t=t_2} \Delta_t F(a,t) + \sum_{t=t_1}^{t=t_2} \Delta_t Z_t(a^1)$$

$$C_t(B_t(S)) \in [c_{min}, c_{max}] \subseteq \Re^P, \forall t>0$$

I.e. the purpose of management on this level is supporting of productive regime, and formal criteria determined by ratio analysis on (6-7).

For this sub-problem general algorithm of management is:

1. Construction of the management program
$$P_t(B_t(S)) = \{B_{t+1}^{FC}(S),...,B_{t+n*\tau}^{FC}(S)\}, n \in N \quad (9)$$
2. Measurement of characteristics
$$Z_t,\ R_t,\ C_t(B_t(S))$$
3. Analysis of characteristics and ratios (6) and comparison with formal criteria (7):
$$\langle C_t(B_t) \subseteq \hat{C}; \Delta_t C \le \Delta^{\max} \rangle \Leftrightarrow P_t(B_t) = const$$
i.e. analysed ratios have to be adequate to normal regime of functioning, and have not sharp changes. The threshold for changes of ratios supposed to be defined by experts;
4. Corrections for the management program in accordance with changing environment and management purpose criteria and measurements (8).

This algorithm realised the management problem solution on the feed-back principle.

For the macro-level of the system:

$$B_{t+1}(Z, F_{t+1}^{0}(Z, R_0); \Phi_{t+1}) = U_t^I(B_t(S_t; \Phi_t)) \quad (10),$$

$$\sum_{t=t_1}^{t=t_2} \Delta_t F(a,t) + \sum_{t=t_1}^{t=t_2} \Delta_t Z_t(a^1) - \sum_{t=t_1}^{t=t_2} \Delta_t Z_t(a^0) =:: Y_t^{t+\tau}$$

$$Y_t^{t+\tau}(B_t(S)) \to \max, \forall t > 0$$

$$P_t(B_t(S)) = \{B_{t+1}^{FC}(S), \ldots, B_{t+n*\tau}^{FC}(S)\}, n \in N$$

I.e. the purpose of management on this level is improving the effectively of productive regime, within the measures of structural integrity supporting, and formal criteria determined in by ratio analysis on (6-7).

For this sub-problem general algorithm of management is:

1. On the solution of sub-problem (8), selection of the management program, aggregated to macro-level of hierarchy;
2. Measurement of quantitative characteristics:

   3. Z_t, R_t, $C_t(B_t(S))$
3. Correction of the management program in accordance with criteria of the sub-problem (10) and solution for sub-problem (8) found. Such correction can result with reduction of uncertainty for full-controlled characteristics, with measurements from (10).

This algorithm also realised the management problem (10) solution on the feed-back principle.

Proposed approach combined two one-purpose management problems solutions into the aggregated solution for general multi-purpose management problem (7).

As the result of modelling for certain system on the base of proposed fuzzy hierarchical model we can calculate the set of management purpose adequate values for the system characteristics. It will be the combination of fuzzy intervals:

$$x_t \in \bigcup_i (x_t^{\min}; x_t^{med}; x_t^{\max}) \subseteq \Re,$$

where x_t - characteristics of S_t we can define scenario as completed set of characteristics of the system S, defined on coherent multi-dimensional interval from the set of values, adequate to measurements of (8) and (9):

For future states of the system we can calculate the set of acceptable scenarios, which have to be corrected with changes of characteristics of the environment:

$$\hat{X}_{t+\tau}(S_{t+\tau}) \in (X_{t+\tau}^{\min}; X_{t+\tau}^{med}; X_{t+\tau}^{\max})$$

On the basis of proposed model we can build the set of classes for fully controlled characteristics and establish relation between criteria (7) and these classes. Thus we can establish relation between solutions of management problem and classes of fully-controlled characteristics of the system. Based on this relation we can transform the solution of management problem to the appropriate solution of pattern recognition problem.

4. Conclusion

On the basis of fuzzy hierarchical model of productive system we proposed approach, which makes solution of the multipurpose management problem more effective. This approach based on the decomposition of multidimensional management purpose to simple purposes. And one-purpose management problems can be solved on different hierarchical levels of the proposed model, and combined to the general solution.

Uncertainty reduction of information for possible alternatives for management system dynamic proposed to perform with scenario analysis. Based on this approach we can recognise in the set of system states the sub-set which is most adequate for management purpose criteria.

On this sub-set of Pareto-optimal scenarios of the system dynamic we can determine proper classes of management actions. I.e. we perform decomposition of the sets of system states and related management actions to interrelated sets of classes. This decomposition allows to reduce multi-purpose management problem for the structured system to the pattern recognition problem.

Thus, the modelling of structured productive system functioning under uncertain influence of the environment can be the base for research of system for multi-purpose management problem solution.

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