

Educational Institution «Belarus State Economic University»

**APPROVED**

Rector for Educational Establishment  
«Belarus State Economic University»

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A.V. Egorov  
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2023 ye.  
Registration № УД 4895-21 /уч.

**HIGHER MATHEMATICS**

The curriculum of the educational institution for the specialty  
1-25 01 08 «Accounting, analysis and audit (by areas)»  
1-25 01 03 «World Economy»  
(in English)

The curriculum is based on the standard curriculum for the academic discipline "Higher Mathematics", approved by the First Deputy Minister of Education of the Republic of Belarus, registration No. ТД- Е.890/тип. from. 11.09.2022.

**COMPILED BY:**

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**RECOMMENDED FOR APPROVAL:**

Department of Higher Mathematics of the educational institution "Belarus State Economic University"

(Protocol №. 10 dated April 19, 2023);

Methodological Commission institutions education « Belarusian state economic university »

(Protocol №. 6 dated 22.06, 2023).

## EXPLANATORY NOTE

The curriculum for the academic discipline “Higher Mathematics” is **aimed** at studying basic mathematical concepts and methods that will allow the future specialist to freely navigate a variety of mathematical models and methods. For specialists in economics and management, mathematics is largely a tool for processing and analyzing information, decision making and management.

As noted by L.D. Kudryavtsev (Selected works. Vol. 3. Thoughts about modern mathematics and its teaching. - M.: Fizmatlit , 2008. - P. 325–326) “...natural science and mathematics education is needed not only to tell students certain information on the subjects being studied, but also because it contributes to the understanding of the laws that govern the world around us, and, therefore, shapes the worldview of students, and therefore is part of humanitarian education, in the broad sense of the word, part of universal human culture, which cannot be filled by studying purely humanitarian disciplines. For example, teaching mathematics aims not only to familiarize students with mathematical concepts and develop skills in their use, but also develops thinking, teaches them to think logically, discard what is not essential for solving the task at hand, develops aesthetic feelings and a sense of self-honesty . So, the study of natural sciences and mathematics is a necessary condition for the correct formation of a student’s full-fledged personality. This goal can only be achieved if a sufficient number of hours are allocated to the subjects of the natural sciences and mathematics, necessary not only to become familiar with the concepts studied in these disciplines, but also to master them by solving a sufficient number of problems . ”

**The purpose** of teaching the discipline “Higher Mathematics” is to familiarize students with mathematical concepts, methods and skills of using them to solve typical applied problems, as well as the development of abstract, logical and algorithmic thinking.

In this regard, when presenting educational material in the academic discipline “Higher Mathematics,” teachers face the following **objectives**:

- considering mathematical culture as part of universal human culture, to contribute to the formation of a highly moral civic position of students, the formation of an integral, highly intellectual personality capable of solving complex problems that life poses;
- give an idea:
  - a) about the place of mathematics in the system of natural sciences and economic sciences;
  - b) about the inextricable unity of applied and fundamental mathematics;
  - c) about the advantages of mathematical modeling and its economic efficiency;
- introduce students to the basic concepts and methods of modern mathematics;
- teach to apply mathematical knowledge when studying real economic processes and solving professional problems;
- to develop students' abilities for abstract and logical thinking ;

- to instill in students the motivation to deeply study mathematics as the language of communication of civilized economists, without which it is impossible master special disciplines they need in their future professional activity.

As a result of studying the academic discipline “Higher Mathematics”, the following basic professional **competence** is formed:

BPC-1. Use basic mathematical concepts and calculation methods to analyze and model economic processes.

As a result of studying the academic discipline, the student are *to know*:

– methods of matrix algebra and analytical geometry, mathematical apparatus of functions of one and many variables, fundamentals of differential equations, numerical and power series, transport problem, mathematical apparatus of linear, integer, nonlinear and dynamic programming;

*to be able to*:

– solve problems of matrix algebra, analytical geometry and mathematical analysis, analyze problems with economic content;

– study optimization problems using mathematical programming methods using computer technologies;

*to possess*:

– methodology for applying the methods of matrix algebra, analytical geometry, differential and integral calculus, mathematical programming in solving mathematical and economic problems.

The academic discipline “Higher Mathematics” belongs to the “Mathematics” **module** of the state component.

**Connection with other academic disciplines:** The material of the academic discipline “Higher Mathematics” is basic for the academic disciplines “Statistics”, “Econometrics”, “Economic Theory”, “Microeconomics”, “Macroeconomics”.

As part of the educational process in this academic discipline, the student must acquire not only theoretical and practical knowledge, skills and abilities in the specialty, but also develop his value-personal, spiritual potential, develop the qualities of a patriot and citizen, ready for active participation in economic, industrial, socio-cultural and public life of the country.

Form of higher education degree program: full-time.

According to the curriculum, the number of hours are the following:

total number of academic hours — 328, in-class — 148 hours, including lectures 74 hours, practical classes — 74 hours.

Distribution of in-class hours by type of classes, years and terms:

1 term - lectures 28 hours, practical classes — 24 hours;

2 term - lectures 28 hours, practical classes — 34 hours;

3 term - lectures 18 hours, practical classes — 16 hours.

Self study – 180 hours.

The complexity of the academic discipline is 9 credit units.

Forms of intermediate certification - tests, exam.

## CONTENT OF TRAINING MATERIAL

**Topic 1. Fundamentals of set theory and mathematical symbolism****Topic 1.1. Elements of set theory**

Elements of set theory and mathematical logic. Logical symbols, operations on sets. Cartesian product of sets. Economic examples. Basic numerical sets. Exact and approximate values of quantities. Absolute and relative errors. Necessary and sufficient conditions. Method of mathematical induction. Binomial theorem.

**Topic 1.2. Complex numbers**

Complex numbers and operations on them. Geometric interpretation of complex numbers. Algebraic, trigonometric and exponential forms of complex numbers. Moivre and Euler formulas. Extracting the root of a complex number. Properties of complex conjugate expressions. Application of complex numbers.

**Topic 2. Vector algebra and matrix calculus****Topic 2.1. Vector algebra**

Concept vectors on the plane and in three-dimensional space. Basic linear operations on vectors. Projection of a vector onto an axis. Linear dependence of vectors. Basis on the plane and in space. Cartesian coordinate system. Radius vector and point coordinates. Division of a segment in this respect. Dot product of vectors, its properties and economic interpretation. Condition for orthogonality of two vectors. Orientation of a triple of vectors in space. Condition for coplanarity of three vectors. Linear spaces. Subspace and linear hull. Basis and dimension of linear space. Euclidean spaces. Vector norm and its properties. Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process.

**Topic 2.2. Matrix calculus**

The concept of a matrix and linear operations on them. Transposing matrices. Matrix trace. Economic interpretation of matrices. Permutations and transpositions. Determinants of the second and third order. Algebraic complements and minors. Determinants of  $n$ th order and their properties. Rules for calculating determinants, Laplace's theorem. Determinant of the product of matrices. Inverse matrix, properties of inverse matrices. Methods for calculating the inverse matrix. Matrix rank, properties and its calculation. The condition for the determinant to be equal to zero. The theorem on the basis minor. Similar matrices. Reducing the matrix to diagonal form. Quadratic forms and their matrices. Reducing a quadratic form to canonical form by an orthogonal transformation. Definite quadratic forms. Conditions for sign definiteness of quadratic forms. Interindustry balance model V.V. Leontyev.

Distinctive features of the Belarusian economic model.

### **Topic 2.3. Systems of linear equations and inequalities**

Systems of linear algebraic equations, general concepts. Economic examples. Theorem Kronecker-Capelli . Matrix method for solving linear systems. Formulas Cramer method Gauss . Homogeneous and inhomogeneous systems of linear equations. Structure of the general solution. The concept of an approximate solution of a system of equations. Decomposition of a vector according to an orthogonal basis. Eigenvectors and eigenvalues of matrices, their properties. Characteristic equations. Eigenvectors and eigenvalues of symmetric matrices. Systems of linear inequalities. Graphical method for solving a system of linear inequalities with two variables. Application of elements of linear algebra in economics. Inter-industry balance model. Distinctive features of the Belarusian economic model.

### **Topic 3. Analytical geometry**

#### **Topic 3.1. Analytical geometry on the plane**

Subject of analytical geometry. Coordinate method. Curve on a plane and methods for defining it. Basic types of equation of a straight line. Angle between straight lines. Conditions for parallelism and perpendicularity of two straight lines. Distance from a point to a line. Second order curves: circle, ellipse, parabola, hyperbola. Parametric representation of lines.

#### **Topic 3.2. Elements of analytical geometry in space**

The simplest problems of analytical geometry in space. Concepts of surface and curve in space, their equations. Basic types of equations of a plane and a line in space. The angle between two straight lines. The angle between a straight line and a plane. Distance from a point to a plane. The concept of second order surfaces.

### **Topic 4. Mathematical analysis**

#### **Topic 4.1. Number sequence and its limit**

Number sequences. The sequence limit and its properties. Infinitely small and infinitely large sequences. Monotonous, limited sequences. Properties of convergent sequences and criteria for their convergence. Methods for calculating sequence limits. The number "  $e$  " and its economic interpretation.

#### **Topic 4.2. Functions of one real variable**

Functions, their domains of definition and values, methods of specifying and

graph of the function. Basic characteristics of the function's behavior. Basic elementary functions. Superposition of functions, inverse functions. Implicit functions. Limit of a function at a point and at infinity. Basic theorems on the limits of functions. Remarkable limits. One-sided limits. Infinitely small and infinitely large functions. Continuity of a function at a point. Properties of continuous functions. One-way continuity. Breakpoints of functions and their classification. Continuity of elementary functions. Comparison of functions, symbols " $o$ " and " $O$ ". Equivalent functions, their application to the calculation of limits of functions. Functions continuous on an interval and their properties: theorems of Weierstrass, Cauchy on the passage of a function through zero, Cauchy on an intermediate value. Continuity of the inverse function. Uniform continuity of a function on a segment.

### **Topic 4.3. Differential calculus of functions of one real variable**

Problems leading to the concept of derivative. Derivative of a function, its geometric, physical and economic meanings. Equation of tangent and normal to a curve. One-sided derivatives. Basic rules of differential calculus. Derivative of complex, inverse and implicit functions. Derivatives of elementary functions. Logarithmic differentiation. Differentiation of parametrically specified functions. Differential of a function, its geometric meaning and application in approximate calculations. Invariance of the shape of the differential. Derivatives of higher orders. Leibniz's formula. Local extremum of a function. Basic theorems of differential calculus: Fermat, Rolle, Cauchy, Lagrange. Application of theorems. Disclosure of uncertainties, L'Hopital -Bernoulli rule. Taylor's formula. Residue term in Peano and Lagrange form. Basic expansions using the Taylor formula. Applications of Taylor's formula. Extrema of a function, stationary points. Necessary and sufficient conditions for a local extremum. The largest and smallest values of a function on a segment. Convexity and inflection points. Sufficient condition for convexity. Necessary condition for inflection. Sufficient conditions for inflection. Vertical and oblique asymptotes of the graph of a function. General scheme for studying a function and constructing its graph. Economic applications: marginal indicators in economics, elasticity of economic indicators, profit maximization.

### **Topic 4.4. Functions of many variables**

Sets on the plane and in space. Limit points of sets. Connected, convex, bounded sets. The concept of a function of several variables, examples from economics. Level lines, isocosts, isoquants. Homogeneous functions. Convex and concave functions. Limit of a function at a point, repeated limits. Continuity. Properties of continuous functions. Partial derivatives. Examples of the use of partial derivatives in economics. Differentiability of a function of several variables, necessary and sufficient conditions for differentiability. Total differential and its connection with partial derivatives. Differentiation of complex functions. Invariance

of the form of a total differential. Directional derivative and its properties. Function gradient and its meaning. Partial derivatives of higher orders. Theorem on the equality of mixed derivatives. Differentials of higher orders. Jacobian, Hessian matrix. Taylor's formula for a function of two variables. The concept of extremum of a function of several variables. Necessary and sufficient conditions for an extremum. Least square method. The largest and smallest values of a function in a given area. Conditional extremum. Lagrange multiplier method. Applications to economic problems.

#### **Topic 4.5. Antiderivative and indefinite integral**

Antiderivative of a function and indefinite integral. Properties of the indefinite integral. Variable replacement method. Formula for integration by parts. Table of indefinite integrals. Integration of simple rational fractions. Integration of rational functions. Integrating expressions containing trigonometric functions. Non-permanent integrals.

#### **Topic 4.6. Definite integral**

Problems leading to the concept of a definite integral. Definite integral. Basic properties of a definite integral. Integrability conditions functions. Integral with a variable upper limit and its differentiation. Newton - Leibniz formula . Change of variable in a definite integral. Integration formula by parts for a definite integral. Geometric applications of definite integrals: calculation of areas of plane figures, volumes of bodies, lengths of arcs, areas of surfaces of revolution. Economic applications of definite integrals. Improper integrals and signs of their convergence.

### **Topic 5. Differential and discrete equations**

#### **Topic 5.1. Ordinary differential equations**

Basic concepts of the theory of ordinary differential equations, general and particular solutions. Mathematical modeling in economics and technology using differential equations. Cauchy problem. Theorem of existence and uniqueness of a solution. First order differential equations and methods of their integration. Linear differential equations of the first and second orders. Homogeneous and inhomogeneous linear differential equations of the second order with constant coefficients and a special right-hand side. Characteristic equation. Linear independence of solutions. Lagrange's method of variation of an arbitrary constant. General concepts about differential equations of higher orders. Applications of differential equations to solving economic problems.

#### **Topic 5.2. Discrete equations**

Discrete (difference) equations. Finite differences. Economic problems leading

to difference equations. General concepts of difference equations. Homogeneous and inhomogeneous linear difference equations and the structure of their general solutions. Solution of linear homogeneous difference equations with constant coefficients. Systems of discrete equations and their properties, methods for finding their solutions.

## **Topic 6. Numerical and power series**

### **Topic 6.1. Number series**

Number series and its sum. Action on rows. The simplest properties of number series. A necessary condition for the convergence of a series. Harmonic series. Tests for the convergence of number series: Cauchy criterion, comparison tests, d'Alembert and Cauchy tests, integral test. Alternating series, absolute and conditional convergence. Alternating series, Leibniz's sign. Estimation of the remainder of the series. Properties of absolutely and conditionally convergent series.

### **Topic 6.2. Power series**

Power series, Abel's theorem. Radius, interval and region of convergence of a power series. Continuity of sums, integration and differentiation of power series. Rows Taylor and Maclaurin. Sufficient conditions for representing a function by a Taylor series. Taylor series expansion of basic elementary functions. Application of Taylor series in approximate calculations.

## **Topic 7. Mathematical Programming**

### **Topic 7.1. Linear programming**

Basic formulations of linear programming problems (LPPs). Geometric (graphical) method for solving LLP. Simplex method for solving LLP. Duality theory. Technology planning problem. The problem of planning production levels. Using the QSBR and EXCEL application package to solve linear programming problems.

### **Topic 7.2. Transport task**

Transport problem according to the cost criterion and transport type problems with maximization and minimization of the objective function. Method of potentials for solving transport problems. Using the QSBR and EXCEL application package to solve transport problems.

### **Topic 7.3. Integer programming**

Statement of integer programming problems: general scheduling problem, traveling salesman problem, partitioning, covering and packaging problems,

equipment placement problem, cutting problem. Branch and bound methods. Cutting methods.

#### **Topic 7.4. Nonlinear programming**

Statement of the nonlinear programming problem and its geometric interpretation. Lagrange multiplier method. Convex and concave functions. Convex programming problems. Kuhn- Tucker theorem . The concept of local and global optimum. Gradient methods for solving nonlinear programming problems. Approximate methods for solving nonlinear programming problems with separable functions. Quadratic programming. Application of application packages for solving nonlinear programming problems.

#### **Topic 7.5. Dynamic programming**

Concept of dynamic programming: Bellman optimality principle, Bellman function. Examples of problems solved using the dynamic programming method. Computational scheme of the dynamic programming method. Dynamic tasks of choosing the most rational route for cargo delivery, optimal allocation of funds for expansion of production, determining the optimal strategy for replacing equipment, forming an optimal production program taking into account inventories. Using the Network package Optimization (network optimization) to select the most economical cargo delivery route.

[illegible]

|                       |                                                         |           |           |  |  |  |  |  |              |                                    |
|-----------------------|---------------------------------------------------------|-----------|-----------|--|--|--|--|--|--------------|------------------------------------|
| Topic 4.3             | Differential calculus of functions of one real variable |           | 4         |  |  |  |  |  | K, [ 1 , 3 ] | Problem solving. CGW               |
| Topic 4.4             | Functions of many variables                             | 6         |           |  |  |  |  |  | K, [2, 4     | Sample survey.                     |
|                       | Functions of many variables                             |           | 6         |  |  |  |  |  | K, [2, 4     | Problem solving.                   |
| Topic 4.5             | Antiderivative and indefinite integral                  | 4         |           |  |  |  |  |  | K, [2, 4 ]   | Sample survey.                     |
|                       | Antiderivative and indefinite integral                  |           | 6         |  |  |  |  |  | K, [2, 4 ]   | Problem solving.                   |
| Topic 4.6             | Definite integral                                       | 6         |           |  |  |  |  |  | K, [2, 4 ]   | Sample survey. Checking the notes. |
|                       | Definite integral                                       |           | 6         |  |  |  |  |  | K, [2, 4 ]   | Problem solving.                   |
| 5                     | <b>Topic 5. Differential and discrete equations</b>     |           |           |  |  |  |  |  |              |                                    |
| Topic 5.1             | Ordinary differential equations                         | 6         |           |  |  |  |  |  | K, [2, 4 ]   | Sample survey.                     |
|                       | Ordinary differential equations                         |           | 6         |  |  |  |  |  | K, [2, 4 ]   | Problem solving.                   |
| 6                     | <b>Topic 6. Numerical and power series</b>              |           |           |  |  |  |  |  |              |                                    |
| Topic 6.1             | Number series                                           | 3         |           |  |  |  |  |  | K, [2, 4 ]   | Sample survey.                     |
|                       | Number series                                           |           | 3         |  |  |  |  |  | K, [2, 4 ]   | Problem solving.                   |
| Topic 6.2             | Power series                                            | 3         |           |  |  |  |  |  | K, [2, 4 ]   | Sample survey.                     |
|                       | Power series                                            |           | 3         |  |  |  |  |  | K, [2, 4 ]   | Problem solving.                   |
| <b>Total 2nd term</b> |                                                         | <b>28</b> | <b>34</b> |  |  |  |  |  |              | <b>Exam</b>                        |
| <b>3rd term</b>       |                                                         |           |           |  |  |  |  |  |              |                                    |
| 7                     | <b>Topic 7. Mathematical Programming</b>                |           |           |  |  |  |  |  |              |                                    |
| Topic 7.1             | Linear programming                                      | 4         |           |  |  |  |  |  | K, [5, 6]    | Sample survey.                     |
|                       | Linear programming                                      |           | 4         |  |  |  |  |  | K, [5, 6]    | Problem solving.                   |
| Topic 7.2             | Transport task                                          | 4         |           |  |  |  |  |  | K, [5, 6]    | Sample survey.                     |
|                       | Transport task                                          |           | 4         |  |  |  |  |  | K, [5, 6]    | Problem solving. CGW               |
| Topic 7.3             | Integer programming                                     | 2         |           |  |  |  |  |  | K, [5, 6]    | Sample survey.                     |
|                       | Integer programming                                     |           | 2         |  |  |  |  |  | K, [5, 6]    | Problem solving.                   |
| Topic 7.4             | Nonlinear programming                                   | 4         |           |  |  |  |  |  | K, [5, 6]    | Sample survey.                     |
|                       | Nonlinear programming                                   |           | 4         |  |  |  |  |  | K, [5, 6]    | Problem solving.                   |
| Topic 7.5             | Dynamic programming                                     | 4         |           |  |  |  |  |  | K, [5, 6]    | Sample survey.                     |
|                       | Dynamic programming                                     |           | 2         |  |  |  |  |  | K, [5, 6]    | Problem solving.                   |
| <b>Total 3rd term</b> |                                                         | <b>18</b> | <b>16</b> |  |  |  |  |  |              | <b>Test</b>                        |
| <b>Total hours</b>    |                                                         | <b>74</b> | <b>74</b> |  |  |  |  |  |              |                                    |

### Literature

#### Main:

- 1 Astrovsky, A. I. Higher mathematics: a textbook for students of higher education institutions in economic specialties: in 2 hours / A. I. Astrovsky, M. P. Dymkov. – Minsk: BSEU, 2022–2023. – Part 1. – 2022. – 415 p.
- 2 Astrovsky, A. I. Higher mathematics: a textbook for students of higher education institutions in economic specialties: in 2 hours / A. I. Astrovsky, M. P. Dymkov. – Minsk: BSEU, 2022–2023. – Part 2. – 2023. – 412 p.
- 3 Collection of problems and exercises in higher mathematics for students of economic specialties: educational and methodological manual: in 2 parts / Ministry of Education of the Republic of Belarus, Belarusian State Economic University; [A. V. Konyukh and others]. – 2nd ed., revised. – Minsk: BSEU, 2021. – Part 1. – 307 p.
- 4 Collection of problems and exercises in higher mathematics for students of economic specialties: 2 parts / Ministry of Education of the Republic of Belarus, EE "Belarusian State Economic University". – Minsk: BSEU, 2008 – 2009. – Part 2 / [L.N. Gaishun and others]. – 2008. – 270 p.
- 5 Kuznetsov, A.V. Higher mathematics: mathematical programming: a textbook for students of economics. specialist. universities / A.V. Kuznetsov, V.A. Sakovich, N.I. Cold; edited by A.V. Kuznetsova. – 2nd ed. – Minsk: Higher School, 2001. – 315, [1] p.: ill.
- 6 Kuznetsov, A.V. Guide to solving problems in mathematical programming: textbook. allowance / A.V. Kuznetsov, N.I. Kholod, L.S. Kostevich; edited by A.V. Kuznetsova. – Minsk: Higher school, 2001. – 448, [1] p.: ill.

#### Additional:

- 7 Higher mathematics: workshop for students of economic specialties of universities: in 2 hours / Ministry of Education of the Republic of Belarus, EE "Belarusian State Economic University". – Minsk: BSEU, 2008 – 2011. – Part 1 / [A. V. Konyukh and others]. – 2008. – 253 p.
- 8 Higher mathematics: workshop for students of economic specialties of universities: in 2 hours / Ministry of Education of the Republic of Belarus, EE "Belarusian State Economic University". – Minsk: BSEU, 2008–2011. – Part 2 / [V. V. Kosyanchuk and others]. – 2011. – 234, [1] p.
- 9 Pismenny, D.T. Collection of problems in higher mathematics: With tests: 2nd year: textbook / D. T. Pismenny. – M.: Iris-press. 2019. – 589 p.
- 10 Ilyin, V. A. Higher mathematics: textbook / V. A. Ilyin, A. V. Kurkina. – 3rd edition. – M.: Prospekt, 2020. – 176 p.

- 11 Shapkin, A. S. Problems with solutions in higher mathematics, probability theory, mathematical statistics, mathematical programming: textbook. allowance / V. A. Shapkin, A. S. Shapkin. – M.: Dashkov and K<sup>o</sup>, 2015. – 432 p.
- 12 Kundysheva , E. S. Mathematics: a textbook for economists / E. S. Kundysheva . – M.: Dashkov and K<sup>o</sup>, 2015. – 562 p.
- 13 Machulis , V.V. Higher mathematics: a textbook for universities / V.V. Machulis . – M.: Yurayt , 2016. – 306 p.
- 14 Collection of problems in higher mathematics: with tests / K. N. Lungu [et al.]. – 10th ed. – M.: Iris-press, 2017 – . – (Higher education). – Part 1: [Linear algebra. Analytic geometry. Fundamentals of mathematical analysis. Complex numbers]. – 2017. – 574, [1] p.
- 15 Mathematics in problems and tasks: Textbook for foreign students of higher education institutions in technical and economic specialties / L.I. Maysenya [and others]. – Minsk: Higher School, 2023. – 557, [1] p.

**List of questions for testing  
in the discipline "Higher Mathematics"  
(1st term)**

1. Basic concepts about vectors (geometric interpretation: free and bound vectors, modulus, angle between vectors, collinearity, coplanarity).
2. Multiplying a vector by a number, sum of vectors.
3. Linear combination of vector system. Linear dependence and independence of vectors. Rank of the vector system. Basis. Decomposition of a vector by basis.
4. Coordinate method. The distance between two points. Cartesian coordinate system in space.
5. Dot product of vectors and its properties.
6. Definition of linear space. Dimension and basis of linear space. Euclidean space.
7. Matrix. Basic operations on matrices.
8. Applications of matrices.
9. Matrix determinant. Properties of the determinant and its calculation.
10. Minors. Algebraic additions. Laplace's theorem.
11. Matrix rank and its determination.
12. Inverse matrix and methods for finding it. Theorem of existence and uniqueness of the inverse matrix.
13. Systems of linear equations. Basic concepts. Matrix recording form.
14. Homogeneous systems of linear equations. Conditions for the existence of a non-zero solution.
15. The concept of an extended matrix of a system of equations. Kronecker-Capelli theorem.
16. Inverse matrix method for solving systems of linear equations.
17. Cramer's rule for solving systems of linear equations.
18. Gauss method for solving systems of linear equations.
19. Applications of systems of linear equations.
20. Various types of equations of a straight line on a plane.
21. Angle between straight lines. Conditions for parallelism and perpendicularity of lines. Distance from a point to a line.
22. Lines of the second order. Equation of a circle.
23. The main stages of deriving the equation of an ellipse, hyperbola, and parabola.
24. Equations of a plane in space. Normal vector.
25. Equations of a straight line in space. Guide vector.
26. The relative position of two lines in space.
27. The relative position of two planes in space.
28. Sets and operations on them. Cartesian product of sets. Basic numerical sets.
29. Definition of a number sequence. Consistency limit.
30. Properties of bounded, monotonic sequences. Examples.
31. Basic properties of convergent sequences.

32. Fundamental sequences. Examples. Convergence criterion.
33. Infinitely small and infinitely large sequences. Examples.
34. The number "e" as the limit of the sequence.
35. Application of the number "e".
36. Methods for specifying functions of one real variable. Domain.
37. Function graph. Complex and inverse functions. Parametrically defined functions.
38. Characteristics of function behavior.
39. Limit of a function at a point. One-sided limits. Continuity of function.
40. Properties of function limits. Elementary techniques for disclosing uncertainties.
41. Two great limits.
42. Discontinuous functions. Types of breaks.
43. Properties of continuous functions. Theorems of Cauchy and Weierstrass.
44. Functions of supply and demand.
45. Definition of a derivative, its physical, geometric and economic meanings.
46. Tangent and normal equations.
47. One-sided derivatives. Condition for the existence of a derivative.
48. Relationship between continuity and differentiability of functions.
49. Infinitesimal functions and their properties. Classification of infinitesimal functions.
50. Definition of the differential of a function, its geometric meaning.
51. Basic rules of differential calculus.
52. Table of derivatives of basic elementary functions.
53. Differentiation of complex functions.

**List of questions for the exam  
in the discipline "Higher Mathematics"  
(2nd term)**

1. Functions of several variables (basic concepts). Limit and continuity of a function of several variables.
2. Partial derivatives of functions of several variables.
3. Total differential of a function of several variables and its application in approximate calculations.
4. Extrema of a function of several variables. Necessary conditions for an extremum. A sufficient condition for the extremum of a function of two variables.
5. The concept of empirical formulas. Selection of parameters using the least squares method. Alignment along a straight line, parabola.
6. Definition of antiderivative function and indefinite integral. Properties of the indefinite integral. Table of basic integrals.
7. Changing a variable (substitution) in an indefinite integral.
8. Integration by parts in an indefinite integral.
9. Integrating some expressions containing a quadratic trinomial.
10. Integration of rational functions.
11. Problem on the area of a curved trapezoid. Definition of a definite integral.
12. Properties of a definite integral.
13. Theorem on the derivative of an integral with a variable upper limit. Newton-Leibniz formula.
14. Theorem on the mean value of a definite integral. Applications in economics.
15. Change of variable and integration by parts in a definite integral.
16. Geometric applications of the definite integral. Area of a flat figure. Volume of a body of revolution.
17. Improper integrals with infinite limits of integration.
18. Improper integrals of unbounded functions.
19. First order differential equations. Cauchy problem. General and particular solution.
20. First order differential equations with separable variables.
21. Linear differential equations of the first order.
22. Linear differential equations of the second order with constant coefficients. Cauchy problem. General and particular solution.
23. Solution of linear homogeneous second order differential equations with constant coefficients.
24. Solution of linear inhomogeneous second order differential equations with constant coefficients.
25. Number series. Convergence of the series. Infinite geometric progression.
26. Properties of convergent series. A necessary condition for the convergence of a series. Harmonic series. Geometric series.

27. A sign for comparing series with positive terms. A sign of comparison in its extreme form.
28. D'Alembert and Cauchy tests for series with positive terms.
29. An integral test for the convergence of series with positive terms. Generalized harmonic series.
30. Alternating series. Cauchy's theorem. Absolute and conditional convergence.
31. Alternating rows. Leibniz's sign.
32. Power series. Interval and region of convergence of a power series. Abel's theorem.
33. Taylor and Maclaurin series . The remainder of the series.
34. Differentiation and integration of power series.

**List of questions for testing  
in the discipline "Higher Mathematics"  
(3rd term)**

1. Subject and tasks of mathematical programming. Economic examples. Statement of the general problem of MP.
2. The LP problem and its various forms. records (general, canonical, symmetric). Converting one form of PPP record to another.
3. Geometric interpretation of the objective function and constraints of the PLP. Geometric formulation of the PLP.
4. Graphical method for solving PLP.
5. Basic plans of the PLP. Correspondence between reference plans and vertices
6. polyhedron of plans.
7. The main theorem of LP. Schematic diagram of the solution to the PLP, which follows from this theorem.
8. Simplex method for solving LPP. General idea of the simplex method. Geometric illustration.
9. A sign of the optimality of the support plan of the PLP.
10. Finding the initial reference plan of the PLP.
11. Finding the optimal reference plan for the PLP.
12. A sign of unboundedness of the objective function on a set of plans and geometric illustration.
13. Sign of infinity of set of optimal plans (alternative optimum) and geometric illustration.
14. Sign of unsolvability of PLP and geometric illustration.
15. Algorithm of the simplex method.
16. The concept of duality in LP. Symmetric dual problems and their economic interpretation. Dual assessments.
17. Asymmetric dual problems. Connections between elements of dual pair problem models. Correspondence between variables of dual problems (dual variables).
18. Duality theorem (basic duality theorem) and its economic interpretation. Finding the optimal plan for a dual problem to solve a straight line.
19. The valuation theorem and its economic interpretation.
20. Properties of duals and their application in the analysis of the solution of the PLP.
21. Formulation and mathematical model of the transport problem based on the cost criterion. Features of the model as a PLP.
22. Transport problem with open and closed model. Converting an open model to a closed one.
23. Condition for the solvability of the transport problem. Integrity conditions for an optimal plan.

- 24.Theorem on the rank of the matrix of the system of constraint equations for the transport problem and its applied significance.
- 25.Cycles in the transport table. Properties of cycles.
- 26.Methods for constructing the initial reference plan of a transport problem (northwest corner, smallest element. Vogel).
- 27.The procedure for converting the reference plan of a transport problem into a new reference plan.
- 28.Evaluation (characteristic) of a free cell in the transport table, its calculation and economic meaning.
- 29.A sign of optimality of the reference plan of a transport problem. Non-uniqueness of the optimal plan.
- 30.Potentials of suppliers and consumers and their calculation.
- 31.Relationship between free cell estimates and potentials.
- 32.Algorithm of the potential method.
- 33.The main inequality of duality theory.
- 34.Existence theorem for optimal plans for a pair of dual problems.
- 35.The first theorem of duality theory.
- 36.The second theorem of duality theory.
- 37.Econometric meaning of dual assessment. Stability interval of dual estimates.
- 38.Theorem on the existence of a plan for a transport problem.
- 39.Theorem on the rank of the matrix of the transport problem.
- 40.Algorithm for constructing a reference plan using the simplex method. Give an example.
- 41.Algorithm for constructing an optimal plan using the simplex method. Give an example.
- 42.Theorem on the choice of a resolving element of a problem solved by the simplex method.
- 43.Theorem on the optimal plan for a problem solved by the simplex method.
- 44.Degeneracy and its elimination when solving problems using the simplex method.
- 45.The case of infinitely many optimal plans. Geometric illustration.
- 46.The case of unboundedness of the objective function on the set of admissible plans of problems solved by the simplex method.
- 47.Statement and mathematical model of the integer linear programming problem. The idea of solving a problem using the cutting method and its geometric illustration.
- 48.Gomori's method for solving a completely integer LP problem.
- 49.Branch and bound method for solving integer problems.
- 50.The concept of dynamic programming. Features of problem solving. Bellman's optimality principle.

- 51. The problem of choosing the shortest path on a road network and solving it using the dynamic programming method.
- 52. Statement of the NLP problem. The concept of convex and concave functions
- 53. Graphical method for solving NLP problems.
- 54. Lagrange method for solving NLP problems.
- 55. Gradient method for solving NLP problems.
- 56. Artificial basis method for solving linear programming problems using the simplex method.

Abbreviations:

- LP – linear programming
- PLP – problem of the linear programming
- MP – mathematical programming
- LPP – linear programming problem
- NLP – non-linear programming

## **Organization of independent work of students**

To obtain competencies in an academic discipline, an important stage is the independent work of students.

A full-time student is allocated 180 hours for independent work.

The main theoretical material is presented in lectures and reinforced in practical classes. Current control is carried out through questioning during practical classes, conducting independent and individual assignments. During each term, one two-hour calculation and graphic work is provided. Final control is carried out in the form of semester tests and an exam.

An important stage in mastering knowledge of an academic discipline is students' independent work. The recommended time budget for independent work is an average of 2-2.5 hours for a 2-hour classroom lesson. The main directions of student's independent work are:

- 1) initially a detailed introduction to the program of the academic discipline;
- 2) familiarization with the list of recommended literature on the discipline as a whole and its sections, its availability in the library and other available sources, studying the necessary literature on the topic, selecting additional literature;
- 3) studying and expanding the teacher's lecture material through special literature and consultations;
- 4) preparation for practical classes with the study of basic and additional literature;
- 5) preparation for performing diagnostic forms of control (calculation and graphic work, tests, colloquiums, tests, etc.);
- 6) preparation for tests and exams.

## **Quality control of knowledge acquisition**

Diagnosis of the quality of knowledge acquisition is carried out within the framework of ongoing monitoring and intermediate certification.

To diagnose competencies in the academic discipline "Higher Mathematics" the following forms can be used: oral, written, oral-written and technical.

Oral forms of competency diagnostics include surveys; reports at practical classes, etc.

The written form of competency diagnostics includes tests, calculation and graphic work, business games, etc.

Oral and written forms of competency diagnostics include presentations, homework reports with their oral defense, etc.

The technical form of competency diagnostics includes electronic tests, etc.

The result of ongoing monitoring for the semester is assessed by a mark on a ten-point scale and is derived based on the marks given during ongoing monitoring activities during the semester.

#### Requirements for students when passing intermediate certification.

Students are allowed to undergo intermediate certification in an academic discipline, subject to successful completion of the current certification (implementation of ongoing control measures) in the academic discipline provided for in the current semester by this curriculum. Interim certification is carried out in the form of tests and exams.

#### **Methodology for forming a grade in an academic discipline**

In accordance with the Regulations on the rating system for assessing the knowledge, skills and abilities of BSEU students.

PROTOCOL FOR COORDINATION OF THE CURRICULUM WITH  
OTHER ACADEMIC DISCIPLINES OF THE SPECIALTY

| Name of the training disciplines, with which approval required | Name departments | Offers about changes in the content of the curriculum higher education institutions education by academic discipline | The decision made by the department that developed the curriculum (indicating the date and protocol number) |
|----------------------------------------------------------------|------------------|----------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|
| 1                                                              | 2                | 3                                                                                                                    | 4                                                                                                           |
| Statistics                                                     | Statistics       | No comments or suggestions<br>     |                                                                                                             |

ADDITIONS AND CHANGES TO THE CURRICULUM FOR THE ACADEMIC  
DISCIPLINE "HIGHER MATHEMATICS"

(Registration No. \_\_\_\_\_ from ... )  
for \_\_\_\_ / \_\_\_\_ academic year

| No.<br>p/p | Additions and changes | Base |
|------------|-----------------------|------|
|            |                       |      |

The curriculum was revised and approved at a department meeting

\_\_\_\_\_ ( Protocol №. \_\_\_\_ dated \_\_\_\_\_ 20\_\_ )  
(name of the department)

Head of the department

\_\_\_\_\_  
(academic degree, academic title)

\_\_\_\_\_  
(signature)

\_\_\_\_\_  
( N. Surname )

APPROVED

Dean of the Faculty

\_\_\_\_\_  
(academic degree, academic title)

\_\_\_\_\_  
(signature)

\_\_\_\_\_  
( N. Surname )